AU–187

B.C.A. (Part-I) Semester-I Examination

DISCRETE MATHEMATICS

Paper—1ST5

Time : Three Hours] [Maximum Marks : 60

Note :— (1) **ALL** questions are compulsory.

(2) Attempt **ONE** question from each unit.

UNIT—I

1. (a) If A and B are finite sets then prove that :

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$ 3

(b) Let $f : A \to B$ and $g : B \to C$. If f and g are one-one then prove that $g \circ f$ is one-one. 3

(c) Amongst the integer 1 to 300, find how many are not divisible by 3 nor by 5. 6

2. (a) Let A and B be countable sets. Then prove that $A \cup B$ is countable. 6

(b) Define permutation and combination with example and prove that :

$${}^{n}c_{r} = {}^{n}c_{n-r}.$$ 6

UNIT—II

3. (a) Find the coefficient of x^{25} in expansion of $(x^3 + x^4 + x^5 + ...)^5$. 6

(b) Define Ferrer's and conjugate Ferrer's diagram and draw both for 9 + 5 + 3 + 2. 6

4. (a) (i) Find the sequence for the exponential generating function for $(1 + x)^{-1}$. 3

(ii) Let A(x), B(x) and C(x) are ordinary generating functions, if $c_n = \alpha\, a_n + \beta\, b_n$ then prove that $C(x) = \alpha\, A(x) + \beta\, B(x)$. 3

(b) Define probability generating function and prove that $E(x) = P'_x(1)$. 6

UNIT—III

5. (a) Solve the Fibonacci relation $a_n = a_{n-1} + a_{n-2}$ with initial conditions $a_0 = 0$, $a_1 = 1$. 6

(b) Find the total solution of $a_r - 6a_{r-1} + 9a_{r-2} = 4$. 6

6. (a) Let a_n be the number of region into which the plane is decomposed by n straight lines. No two of which are parallel, no three of which are concurrent. Find the recurrence relation for a_n. 6

(b) Find the homogeneous solution of the recurrence relation :

$$4a_r - 20a_{r-1} + 17a_{r-2} - 4a_{r-3} = 0.$$ 6

UNIT—IV

7. (a) State Duality Principle. Find the duals of the following :

(i) $(a \wedge b) \geq \bar{b}$

(ii) $(a \vee b) = \overline{(b \vee a)}$

(iii) $a \geq \overline{(b \vee c)}$

(iv) $\overline{(a \vee b)} = 1$

(v) $a \geq 0$

(vi) $\bar{a} \leq 0$. 6

(b) Define Partial order set and prove that $(\rho(A), \subseteq)$ is Partial order set. 6

8. (a) Test whether following is Tautology or contradiction :

(i) $(p \wedge q) \rightarrow (p \vee q)$

(ii) $(p \wedge q) \wedge (\sim p \vee \sim q)$

(iii) $p \wedge (p \rightarrow q)$ 6

(b) Prove that :

$$(a \wedge b) \wedge c \equiv a \wedge (b \wedge c).$$ 6

UNIT—V

9. (a) Prove that in distributive lattice if an element has a complement then this complement is unique. 6

(b) In any Boolean algebra, show that $(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a)$. 6

10. (a) In a Lattice if $a \leq b$ and $c \leq d$, then prove that :

(i) $a \wedge c \leq b \wedge d$

(ii) $a \vee c \leq b \vee d$

(iii) $a \vee o = a$ 6

(b) Let B is the set of statements from closed under $\wedge$, $\vee$ and $\sim$. Show that $< B, \wedge, \vee, \sim, C, t >$ is Boolean algebra, where C is contradiction and t is tautology. 6