

AU - 2534

Third Semester B. E. (Computer Science and Engineering) (CGS) Examination

MATHEMATICS - III

Paper - 3 KS 01

(USC - 10303)

P. Pages : 4

Time : Three Hours]

[Max. Marks : 80

- Note :** (1) Separate answer book must be used for each section in the subject Geology, Engineering material of civil branch and Separate answer book must be used for Section A and B in Pharmacy and Cosmetic Tech.
(2) Due credit will be given to neatness and adequate dimensions.
(3) Assume suitable data wherever necessary.
(4) Illustrate your answer wherever necessary with the help of neat sketches.
(5) Use of calculator is permitted.
(6) Use pen of Blue/Black ink/refill only for writing the answer book.

SECTION A

1. (a) Solve $\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = e^{e^x}$ 6

(b) Solve by the method of variation parameters

$$\frac{d^2y}{dx^2} - y = \frac{2}{1 + e^x} \quad 7$$

OR

2. (a) Solve $(D^2 + 3D + 2)y = 4 \cos^2 x$ 6

(b) Solve $(2x + 1)^2 \frac{d^2y}{dx^2} - 2(2x + 1) \frac{dy}{dx} - 12y = 6x$ 7

3. (a) Find Laplace transform of $\frac{\cos bt - \cos at}{t}$. Hence evaluate

$$\int_0^{\infty} \frac{\cos bt - \cos at}{t} dt. \quad 5$$

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- (b) Using convolution theorem.

$$\text{Find } L^{-1} \left\{ \frac{1}{(s+2)(s^2+16)} \right\}$$

4

- (c) Find Laplace transform of periodic function

$$f(t) = \frac{t}{a}, \quad 0 < t < a$$

$$= \frac{1}{a} (2a - t), \quad a < t < 2a$$

$$\text{and } f(t) = f(t + 2a)$$

5

OR

4. (a) Find $L^{-1} \left\{ \log \left(\frac{s+b}{s+a} \right) \right\}$

4

- (b) Express the following function in terms of Heaviside's unit step function and hence find its Laplace transform, if

$$f(t) = \begin{cases} \cos t & , 0 < t < \pi \\ \sin t & , t > \pi \end{cases}$$

5

- (c) Solve $(D+1)y = t^2 e^{-t}$ by using Laplace transform method, provided $y(0) = 3$

5

5. (a) Solve the difference equation

$$y_{n+2} - 16y_n = \cos \left(\frac{n}{2} \right)$$

5

- (b) Prove that, $z^{-1} \left\{ \frac{z}{z-a} \right\} = a^k$.

4

- (c) Find z - transform of $\sin(3k+5)$

4

OR

6. (a) Solve the difference equations :

(i) $y_{n+2} - 7y_{n+1} + 10y_n = 12e^{3n} + 4^n$

4

(ii) $y_{n+3} - 2y_{n+2} - 5y_{n+1} + 6y_n = 0$

4

(b) Solve by z-transform method

$$y_{n+2} - 5y_{n+1} + 6y_n = 6n,$$

$$\text{if } y(0) = y(1) = 0$$

5

SECTION B

7. (a) Find the Fourier transform of :

$$f(x) = 1, |x| \leq 1$$

$$= 0, |x| > 1$$

$$\text{Hence evaluate } \int_0^{\infty} \frac{\sin x}{x} dx$$

5

(b) Solve the following partial differential equations.

$$(i) p^2 - pq = 1 - z^2$$

4

$$(ii) p^2 - q^2 = x - y$$

4

OR

8. (a) Express $f(x) = 1, 0 \leq x \leq \pi$

$$= 0, x > \pi$$

as a Fourier sine integral and hence evaluate :

$$\int_0^{\infty} \frac{1 - \cos \pi \lambda}{\lambda} \sin \lambda x d\lambda.$$

5

(b) Solve the following partial differential equations :

$$(i) yzp + xzq + 2xy = 0$$

4

$$(ii) p^2 + q^2 = \frac{3a^2}{z^2}$$

4

9. (a) Find the analytic function

$$f(z) = u + iv, \text{ if } \frac{u}{v} = \cot y.$$

5

(b) Expand the function $\frac{e^{2z}}{(z-1)^3}$ about the point $z = 1$ 4

(c) If $f(z)$ is analytic then show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \log |f(z)| = 0 \quad 4$$

OR

10. (a) If $f(z) = u + iv$ is analytic function and

$$(u-v) = (x-y)(x^2 + 4xy + y^2); \text{ find } f(z) \text{ in terms of } z. \quad 6$$

(b) Expand $f(z) = \frac{1}{z^2 - 3z + 2}$ in the region

$$(i) |z| < 1, (ii) 1 < |z| < 2. \quad 7$$

11. (a) Find the directional derivative of $\phi = xyz$ in the direction normal to the surface $x^2y + yz^2 = 3$ at the point $P(1, 1, 1)$. 7

(b) Compute the divergence and curl of vector :

$$\vec{F} = \nabla(yz + zx + xy). \quad 7$$

OR

12. (a) Show that

$$\vec{F} = (x^2 - yz) \mathbf{i} + (y^2 - zx) \mathbf{j} + (z^2 - xy) \mathbf{k} \text{ is irrotational. Hence find a scalar function } \phi \text{ such that } \vec{F} = \nabla \phi. \quad 7$$

(b) Evaluate the line integral $\oint_C \vec{F} \cdot d\vec{r}$, where C is the circle $x^2 + y^2 = 1$ in xy - plane and $\vec{F} = (2x^2 - y^2) \mathbf{i} + (x^2 + y^2) \mathbf{j}$. 7

