

AU – 2521

Third Semester B. E. (Elect. Engg./Elec. and Power) Examination

MATHEMATICS – III

Paper – 3 EP 01 / 3 EX 01 / 3 EL 01 / 3 EE 01

(USC – 10477)

P. Pages : 3

Time : Three Hours]

[Max. Marks : 80

- Note :** (1) Assume suitable data wherever necessary.
(2) Use of calculator is permitted.
(3) Use pen of Blue/Black ink/refill only for writing the answer book.

1. (a) Solve $(D^2 + 5D + 6)y = e^{-2x} \sec^2 x (1 + 2 \tan x)$. 6

(b) Solve

$$(1+x)^2 \frac{d^2 y}{dx^2} + (1+x) \frac{dy}{dx} + y = \sin \{ 2 \log (1+x) \}$$
 7

OR

2. (a) Solve

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = x e^x \sin x$$
 6

(b) Solve by variation of parameter

$$(D^2 + D)y = (1 + e^x)^{-1}$$
 7

3. (a) Find Laplace Transform of $t \cdot e^{3t} \cdot \sin 2t$. 5

(b) Find inverse Laplace Transform of $\frac{e^{-2s}}{s^4}$ 4

(c) Find Laplace Transform of square wave function of period $2a$, defined by

$$f(t) = \begin{cases} 1, & 0 < t < a \\ -1, & a < t < 2a \end{cases}$$
 5

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OR

4. (a) Find inverse Laplace Transform of $\tan^{-1}\left(\frac{2}{S^2}\right)$ 5
- (b) If $L\{J_0(t)\} = \frac{1}{\sqrt{1+S^2}}$, show that $\int_0^{\infty} t \cdot e^{-3t} \cdot J_0(4t) \cdot dt = \frac{3}{125}$ 5
- (c) If $L\{f(t)\} = F(s)$, prove that $L\left\{\int_0^t f(u) du\right\} = \frac{F(s)}{s}$ 4
5. (a) Solve by Laplace Transform method
 $(D+1)^2 y = \sin t; y(0) = 0, y'(0) = 0$ 6
- (b) Find Fourier cosine transform of e^{-x^2} 7

OR

6. (a) Solve by Laplace Transform method
 $(D^3 - 2D^2 + 5D)y = 0; y(0) = 0, y'(0) = 0, y''(0) = 1$ 6
- (b) Solve
 $\int_0^{\infty} f(x) \cdot \sin xt \cdot dx = \begin{cases} 1, & 0 \leq t < 1 \\ 2, & 1 \leq t < 2 \\ 0, & t \geq 2 \end{cases}$ 7
7. (a) Solve $y_{n+1} - 3y_n = n^2 \cdot 2^n$ 4
- (b) Solve $y_{n+2} - 3y_{n+1} - 4y_n + 6 = 0$ 4
- (c) Find inverse Z-transform of $\frac{z}{(z-2)(z+3)^2}$ 5

OR

8. (a) Solve $y_{n+2} - 2y_{n+1} + 2y_n = \cos \frac{n\pi}{2}$ 4
- (b) Find Z-Transform of $K a^{k-1}; k \geq 1$ 1

(c) Solve by using Z-Transform

$$y_{n+2} + 3y_{n+1} + 2y_n = 1; y_0 = 0 \text{ and } y_n = 0, \text{ when } n < 0 \quad 5$$

9. (a) Find the directional derivative of $\Phi = 4e^{2x-y+z}$ at point $(1, 1, -1)$ in the direction towards point $(-3, 5, 6)$. 6

(b) If $\vec{F} = 3xy\vec{i} + 4yz^2\vec{j} - 5xzk$ and $\phi = y^2 - zx$ find $\text{Div}(\Phi\vec{F})$ and $\text{curl}(\Phi\vec{F})$. 8

OR

10. (a) Prove that $\nabla_0(r^3\vec{r}) = \sigma r^3$ 4

(b) If $\vec{r} = xi + yj + zk$ and $\Phi = x^3 + y^3 + z^3 - 3xyz$ find $\vec{r}_0 \cdot \nabla\Phi$. 5

(c) Find the angle between normals to the surfaces $x^2y + z = 3$ and $x \log z - y^2 = -4$ at point $P(-1, 2, 1)$. 5

11. (a) Evaluate $\oint_C \vec{F}_0 \cdot d\vec{r}$ where $\vec{F} = x^2\vec{i} + xy\vec{j}$ and C in the boundary of the square in XY -plane bounded by lines $x=0, x=a, y=0, y=a$. 7

(b) Express $\iiint_S \{(x^3 - yz^2)\vec{i} - 2x^2y\vec{j} + 2k\} \cdot d\vec{s}$ as a volume integral and evaluate it for the volume of a cube with edges of length a parallel to the co ordinate axes. 6

OR

12. (a) If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, then show that $\vec{F} = r^2\vec{r}$ is conservative and find the scalar potential Φ such that $\vec{F} = \nabla\Phi$. 7

(b) If $\vec{F}_1$ and $\vec{F}_2$ are irrotational, show that $\vec{F}_1 \times \vec{F}_2$ is solenoidal. 6

