

B.E. Fourth Semester (Elect. Engineering (Elect. & Power Elect.)) (CGS)
10545 : Mathematics - IV : 4 EP 04 / 4 EL 04 / 4 EE 04

P. Pages : 3

Time : Three Hours



AU - 2583

Max. Marks : 80

- Notes :
1. All question carry marks as indicated.
 2. Answer three question from Section A and three question from Section B.
 3. Use of pen Blue/Black ink/refill only for writing the answer book.
 4. Use of calculator is permitted.

SECTION - A

1. a) Show that the function $u = e^x (x \cos y - y \sin y)$ is harmonic find its harmonic conjugate & the corresponding analytic function $f(z) = u + iv$. 7
- b) If $f(z)$ is analytic function of z then prove that 7
- $$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^n = n^2 |f(z)|^{n-2} \cdot |f'(z)|^2$$

OR

2. a) Find bilinear transformation which maps $\infty, i, 0$ from z -plane into the points $0, i, \infty$ from w -plane. 7
- b) Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ into a straight line $4u+3=0$. 7
3. a) Evaluate $\int_C \frac{\sin^2 z}{(z-\pi/6)^3} dz$ by Cauchy's integral formula where C is the circle $|z|=1$ 6
- b) Find Taylor's and Laurent's series which represents the function $\frac{z^2-1}{(z+2)(z+3)}$ 7
- when i) $|z| < 2$
ii) $2 < |z| < 3$

OR

4. a) If $|z+1| < 1$, show that $z^{-2} = 1 + \sum_{n=1}^{\infty} (n+1) (z+1)^n$ 6
- b) Evaluate the integral by Residue theorem $\oint_C \frac{2z-1}{z(z+1)(z-3)} dz$ where $C : |z|=2$. 7

5. a) Solve $p^2 - pq = 1 - z^2$ 4
 b) Solve $p^2 - y^3 q = x^2 - y^2$ 5
 c) Solve $(D^2 - DD')z = \cos x \cdot \cos 2y$

OR

6. a) Solve $x^2 p^2 + y^2 q^2 = z$ 4
 b) Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$ 4
 c) Solve : $(D^3 - yDD'^2 - 6D'^3)z = e^{2x+y}$ 5

SECTION - B

7. a) Prove that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}} \left[\frac{3-x^2}{x^2} \sin x - \frac{3}{x} \cos x \right]$ 5
 b) Prove that $\int_0^\infty e^{-bx} J_0(ax) dx = \frac{1}{\sqrt{a^2 + b^2}}$ 5
 c) Show that $\int_{-1}^1 x^2 p_{n-1} p_{n+1} dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$ 4

OR

8. a) Prove that $\int x^{2n+1} J_n^2 dx = \frac{x^{2n+2}}{4n+2} [J_n^2 + J_{n+1}^2]$ 5
 b) Show that $p_n(-x) = (-1)^n p_n(x)$ 5
 c) Use Rodrigue's formula to prove $(n+1)p_n = p'_{n+1} - x p'_n$ 4
9. a) Two cards are drawn in succession from a pack of 52 cards. Find the chances that the first is a king and the second is a queen if the first card is 7
 i) replaced
 ii) not replaced
 b) In a sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2 out of 1000 such samples, how many would be expected to contain at least 3 defective parts. 6

OR

10. a) In a bolt factory, machines A, B and C manufactures 25%, 35% and 40% of the total. Out of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B? 7
- b) If the probability that an individual suffers a bad reaction from a certain injection is 0.001. Determine the probability that out of 2000 individuals
- Exactly 3,
 - More than 2 individuals suffer a bad reaction

11. a) Find coefficient of correlation for the following data. 7

x	1	2	3	4	5	6	7	8	9	10
y	10	12	16	28	25	36	41	49	40	50

- b) Verify Sylvester's theorem for A^3 where $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ 6

OR

12. a) Find the equations of lines of regression for the data. 7

x	1	2	3	4	5
y	3	1	2	5	4

- b) Fit a straight line to the data using method of least square. 6

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15
