

AU - 2507

Third Semester B. E. (Electronics and Telecommunication) (CGS) Examination

MATHEMATICS - III

Paper - 3 XT 01/3 IE 01/3 XN 01

(USC - 10591)

P. Pages : 5

Time : Three Hours]

[Max. Marks : 80

- Note :** (1) Answer **Three** questions from Section A and **Three** questions from Section B.
(2) Use of calculator and Normal distribution table is permitted.
(3) Use pen of Blue/Black ink/refill only for writing the answer book.

SECTION A

1. (a) Solve :

$$(D^2 + 5D + 6)y = e^{-2x} \cdot \sec^2 x (1 + 2 \tan x) \quad 7$$

- (b) Solve by method of variation of parameters :

$$(D^2 - 1)y = (1 + e^{-x})^{-2} \quad 6$$

OR

2. (a) Solve :

$$(D^2 - 2D + 4)^2 y = x e^x \cdot \cos(\sqrt{3}x + \alpha) \quad 7$$

- (b) Solve :

$$(x^3 D^3 + x^2 D^2 - 2)y = x + \frac{1}{x^3} \quad 6$$

3. (a) Use fundamental definition, to find Laplace transform of :

$$\begin{aligned} f(t) &= \sin 2t, \quad 0 < t < \pi \\ &= 0; \quad t > \pi \end{aligned} \quad 4$$

AU-2507

P.T.O.

- (b) Find inverse Laplace transform of

$$\frac{S^2 - 3}{(S+2)(S-3)(S^2+2S+5)}$$

5

- (c) Solve by Laplace transform

$$(D^2 + 2D + 1)y = 3te^{-t}$$

$$y(0) = 4, y'(0) = 2$$

5

OR

4. (a) Obtain the inverse Laplace transform of :

$$\frac{e^{-\pi s}}{S^2+4}$$

4

- (b) Find the Laplace transform of periodic function :

$$f(t) = \frac{kt}{T}, 0 < t < T$$

$$\text{and } f(t) = f(t + T).$$

5

- (c) Solve the simultaneous equations :

$$\frac{dx}{dt} = 2x - 3y$$

$$\frac{dy}{dt} = y - 2x$$

$$\text{Subject to } x(0) = 8, y(0) = 3$$

5

5. (a) Solve the following difference equations :

$$(i) u_{n+2} - 2u_{n+1} + 4u_n = 6$$

4

$$(ii) y_{n+2} - 4y_n = n^2 + n - 1$$

4

- (b) Find z-transform of :

$$\frac{(n+1)(n+2)}{2!} \cdot a^n$$

5

OR

6. (a) Solve the difference equation :

$$8y_{n+2} - 6y_{n+1} + y_n = 5 \sin \frac{n\pi}{2} \quad 5$$

- (b) Find z-transform of :

$$\sin(3n+5) \quad 3$$

- (c) Solve by z-transform

$$y_{n+2} + 2y_{n+1} + y_n = n$$

given that $y_0 = y_1 = 0$ 5

SECTION B

7. (a) Using fourier integral, show that

$$\int_0^{\infty} \frac{\sin \pi \lambda \cdot \sin \lambda x}{1-\lambda^2} d\lambda = \frac{\pi}{2} \sin x, \quad 0 \leq x \leq \pi$$

$$= 0 \quad x > \pi \quad 5$$

- (b) Solve

$$x \frac{\partial z}{\partial y} = y \frac{\partial z}{\partial x} + x \cdot e^{x^2+y^2} \quad 4$$

- (c) If the probability that the individual suffers a bad reaction from a certain injection is 0.001. Determine the probability that out of 2000 individuals:

(i) Exactly 3

(ii) More than 2

Individuals will suffer a bad reaction. 5

OR

8. (a) Find the fourier sine transform of e^{-x} , $x \geq 0$ and show that :

$$\int_0^{\infty} \frac{x \cdot \sin mx}{(x^2+1)} dx = \frac{\pi}{2} \cdot e^{-m}, \quad m > 0 \quad 6$$

(b) Solve :

$$pq = x^m y^n z^2$$

4

(c) Fit a Binomial distribution for the given data :

$$x : 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$$

$$y : 2 \quad 14 \quad 20 \quad 34 \quad 22 \quad 8$$

4

9. (a) If $f(z)$ is analytic function with constant amplitude then prove that $f(z)$ is constant. 6

(b) Under the transformation $w = \frac{z-1}{z+1}$ show that the mapping of the straight line $x = y$ is a circle and find its centre and radius. 7

OR

10. (a) Expand :

$$\frac{z^2 - 6z - 1}{(z-1)(z-3)(z+2)}$$

$$\text{in } 3 < |z+2| < 5$$

7

(b) Evaluate :

$$\oint_c \frac{\cos \pi z^2}{(z-1)(z-2)} dz ; c : |z| = 3$$

6

11. (a) Find the directional derivative of $f(x, y, z) = x^2 y^2 z^2$ at the point $(1, 1, -1)$ in the direction of the tangent to the curve $x = e^t$, $y = 2 \sin t + 1$, $z = 1 - \cos t$ at $t = 0$ 6

(b) Use Divergence theorem to evaluate

$$\iiint_S \vec{F} \cdot d\vec{s}, \text{ where } \vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k} \text{ and } S \text{ is the surface bounding}$$

$$\text{the region } x^2 + y^2 = 4, z = 0 \text{ and } z = 3.$$

7

OR

12. (a) If $\phi_1 = x + y + z$, $\phi_2 = x + y$ and $\phi_3 = -(2xz + 2yz + z^2)$

Show that :

$$\nabla \phi_1 \cdot \{ \nabla \phi_2 \times \nabla \phi_3 \} = 0$$

6

- (b) Show that the vector field

$$\vec{F} = (y \sin z - \sin x) \hat{i} + (x \sin z + 2yz) \hat{j} + (xy \cos z + y^2) \hat{k}$$

is irrotational and find the corresponding scalar ϕ such that $\vec{F} = \nabla \phi$. 7

