

AU – 2498

Third Semester B. E. (Mechanical) Examination

ENGINEERING MATHEMATICS - III

3 ME 01

(USC – 10827)

P. Pages : 4

Time : Three Hours]

[Max. Marks : 80

- Note :** (1) Separate answer book must be used for each section in the subject Geology, Engineering material of civil branch and Separate answers book must be used for Section A and B in Pharmacy and Cosmetic Tech.
(2) Answer **Three** questions from Section A and **Three** questions from Section B.
(3) Due credit will be given to neatness and adequate dimensions.
(4) Assume suitable data wherever necessary.
(5) Retain the construction lines.
(6) Use pen of Black ink/refill only for writing the answer book.

SECTION A

1. (a) $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 4y = 8x^2 e^{2x} \sin 2x$ 6
(b) Solve using method of variation of Parameters
 $\frac{d^2y}{dx^2} + y = \sec x$ 7

OR

2. (a) Solve diff. eqn.
 $x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 3x - 7$ 7
(b) $\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = e^{e^x}$ 6
3. (a) Find LT of $\frac{d}{dt} \left(\frac{\sin t}{t} \right)$ 4

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- (b) Find I.T of $\sin t$ and hence show that

$$\int_0^{\infty} \frac{\sin t}{t} dt = \frac{\pi}{2} \quad 4$$

- (c) Express the function in terms of heaviside unit step function and hence find its L. T.

$$\begin{aligned} f(t) &= \cos t, \quad 0 < t < \pi \\ &= \sin t, \quad t > \pi \end{aligned} \quad 5$$

OR

4. (a) Find L^{-1} of

$$\frac{S^2 + 2S + 3}{(S^2 + 2S + 2)(S^2 + 2S + 5)} \quad 5$$

- (b) Verify initial and final value th^m

$$f(t) = e^{-t} (1 + t)^2 \quad 4$$

- (c) Find L^{-1} of $\overline{f(s)} = \frac{s e^{s/2} + \pi e^s}{s^2 + \pi^2}$ 4

5. (a) Solve partial diff. eqn.

(i) $Px \tan y = q + 1$

(ii) $z(p-q) = z^2 + (x + y)^2$ 6

- (b) Fit a second degree parabola to the data

x :	1	2	3	4	5	6	7	8	9
y :	2	6	7	8	10	11	11	10	9

4

- (c) A manufacturer knows that the condensers he makes contain on an average 1% defective. He packed them in boxes of 100. What is the probability that a box picked at random will contain 3 or more faulty condensers ? 4

OR

6. (a) Solve the partial diff. eqn.

(i) $p^2 + q^2 = x + y$

(ii) $pq = x^m y^n z^{2l}$ 6

- (b) Find co-efficient of correlation betⁿ x and y

$$\begin{array}{cccccccc} x : & 1 & 3 & 4 & 6 & 8 & 9 & 11 & 14 \\ y : & 1 & 2 & 4 & 4 & 5 & 7 & 8 & 9 \end{array}$$
4
- (c) The probability that an entering student will graduate is 0.4. Determine the probability that out of 5 students
 (i) none (ii) one (iii) at least one will be graduate. 4

SECTION B

7. (a) Find analytic function $f(z) = u + iv$ if
 $u - v = (x-y)(x^2 + 4xy + y^2)$ 5
- (b) Expand as a Taylors or as a Laurentz series

$$f(z) = \frac{z}{(z-1)(z-2)}$$
 in
 (i) $1 < |z| < 2$
 (ii) $|z-1| > 2$
 (iii) $0 < |z-2| < 1$ 5
- (c) Find the bilinear transformation which maps the points $z = -1, 0, 1$ in to $w = 0, 1, 3i$ in w plane. 4

OR

8. (a) Show that the given function is harmonic. Find corresponding analytic function. Also find its harmonic conjugate
 $u = x^2 - y^2 - 2xy - 2x - y - 1$ 5
- (b) Evaluate using Cauchy's integral formula or by th^m

$$\oint_c \frac{\cos \pi z^2}{(z-1)(z-2)} dz$$
 where c is $|z| = 4$ 5
- (c) Show that the transformation $w = z + \frac{1}{z}$ maps the circle $|z| = c$ in to ellipse
 $u = (c + \frac{1}{c}) \cos \theta, v = (c - \frac{1}{c}) \sin \theta$. 4

9. (a) If $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$

Using Taylors series method find $y(1)$ and $y(2)$.

7

(b) Find root of $x^3 - 2x - 5 = 0$

Using false position method.

6

OR

10. (a) If $\frac{dy}{dx} = \frac{1}{10} (x^2 + y^2)$, $y(0) = 1$

Using Runge-Kutta 4th order method find values of $y(2)$ and $y(4)$.

7

(b) Using Newtons-Raphsons method find root of $2x - \log x = 6$

6

11. (a) The temp. of a point in a space is given by $T = x^2 + y^2 - z$. A mosquito located at (1, 1, 2) desires to fly in such a direction it will warm as soon as possible. In what direction should it move ?

7

(b) Prove that :

(i) $\text{div} \left(\frac{\vec{r}}{r^3} \right) = 0$

(ii) $\text{div}(\text{grad } r^n) = n(n+1) r^{n-2}$

6

OR

12. (a) Show that the vector field $\vec{F} = (2xz^3 + 6y) \mathbf{i} + (6x - 2yz) \mathbf{j} + (3x^2z^2 - y^2) \mathbf{k}$ is irrotational but not solenoidal. Also find scalar potential ϕ such that $\vec{F} = \nabla \phi$.

6

(b) Evaluate $\iint_S \vec{F} \cdot \hat{n} \, ds$ over the entire surface of the region above xy plane bounded by the cone $z^2 = x^2 + y^2$ and the plane $z = 4$ If $\vec{F} = 4xzi + xyz^2j + 3zk$.

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