B.Tech. Fourth Semester (Chem / Poly / Food / Pulp / Oil / Petro) (Old)

Applied Mathematics - III : 4 SCE 1

P. Pages : 3 AU - 3115

Time : Three Hours Max. Marks : 80

Notes :
1. Answer **three** question from Section A and **three** question from Section B.
2. Due credit will be given to neatness and adequate dimensions.
3. Assume suitable data wherever necessary.
4. Illustrate your answer necessary with the help of neat sketches.
5. Use of calculator is permitted.
6. Use of pen Blue/Black ink/refill only for writing the answer book.

SECTION - A

1. a) Solve 6

$$\left(D^4 + 2D^3 - 3D^2\right) y = x^2 + 3e^{2x}.$$

b) Solve 7

$$\left(x^2D^2 + 5xD + 3\right)y = \frac{\log x}{x^2}.$$

OR

2. a) Solve 7

$$(2+3x)^2 \frac{d^2y}{dx^2} + 3(2+3x)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1.$$

b) Solve 6

$$\left(D^2 - 5D + 6\right)y = x\cos 2x.$$

3. a) Find the Laplace transform of 5

$$e^{-4t}\int_0^t t\sin 3t\,dt.$$

b) Find 4

$$L^{-1}\left\{\frac{1}{2}\log\left(\frac{s^2+b^2}{s^2+a^2}\right)\right\}.$$

c) Find the Laplace transform of 5

$$\begin{aligned} f(t) &= (t-a)^4, && t > a \\ &= 0, && 0 < t < a. \end{aligned}$$

OR

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4. a) Find the Laplace transform of f (t) if $f(t)=f(t+a)$, 7

$$f(t)=1, \qquad 0<t<\frac{a}{2}$$
$$=-1, \qquad \frac{a}{2}<t<a.$$

b) Use convolution theorem to find 7

$$L^{-1}\left\{\frac{s^2}{\left(s^2+4\right)^2}\right\}.$$

5. a) Solve the differential equation by using Laplace transform. 6

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=e^{-t}\sin t.$$

with $y(0)=0,\ y'(0)=1.$

b) Find the Fourier sine transform of $\frac{e^{-an}}{x}$. 7

OR

6. a) Solve by using Laplace transform 7

$$\frac{dx}{dt}-y=e^{t}$$
$$\frac{dy}{dt}+x=\sin t$$

with $y(0)=x(0)=1.$

b) Express the function. 6

$$f(x)=1, \qquad |x|\le 1$$
$$=0, \qquad |x|>1$$

as a Fourier integral & hence evaluate

evaluate $\int_0^{\infty}\frac{\sin\lambda\cos\lambda x}{\lambda}d\lambda.$

SECTION – B

7. a) Solve the following difference equations.

i) $U_{x+2}-7U_{x+1}+12U_x=\cos x.$ 4

ii) $Y_{n+2}-4Y_n=9n^2.$ 4

b) Find the inverse Z-transform of 5

$$\frac{3z^2+2z+1}{z^2+3z+2}.$$

OR

8. a) Solve the difference equation 7
$Y_{n+2} + 3Y_{n+1} + 2Y_n = 4^n$.
with $y(0)=0,\ y(1)=1$ using z-transform.

b) Find the z-transform of

i) $c^{3t} \sin t$ 3

ii) $\dfrac{1}{(n+1)!}$ 3

9. a) A particle moves along the curve $x = 2t^2,\ y = t^2 - 4t,\ z = 3t - 5$ where t is the time. Find the components of velocity and acceleration at time $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. 6

b) Find the directional derivative of $\phi = x^2y + y^2z + z^2x$ at (3, 1, 2) in the direction of normal to the surface $xyz = 6$ at (1, 2, 3). 7

OR

10. a) Prove that 6

i) $\text{grad}\, r = \dfrac{\overline{r}}{r}$

ii) $\text{grad}\, r^n = nr^{n-2}\overline{r}$

iii) $\text{grad}\, r = -\dfrac{\overline{r}}{r^3}$

b) Find $\nabla\phi$ if $\phi = 3r^2 - 4\sqrt{r} - \dfrac{2}{\sqrt{r}}$. 7

11. a) If $\overline{F} = 3yx\hat{i} - y^2\hat{j}$, evaluate $\int_C \overline{F}\cdot d\overline{r}$ where C is the curve in xy plane, $y = 2x^2$ from (0, 0) to (1, 2). 7

b) Using Stoke's theorem to evaluate $\oint_C (yz\,dx + zx\,dy + xy\,dz)$ where C is the curve $x^2 + y^2 = 1,\ z = y^2$. 7

OR

12. a) Verify Gauss divergence than for $\overline{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ taken over the rectangular-parallelepiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$. 7

b) Prove that $\overline{F} = f(r)\overline{r}$ is irrotational & find ϕ such that $\overline{F} = \nabla\phi$. 7

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