AP–2769

B.Sc. Semester-I (Old) Examination
MATHEMATICS-I
(Algebra and Trigonometry)

Time—Three Hours] [Maximum Marks—60

N.B. :— Question No. **1** is compulsory and attempt **ONE** question from each unit.

1. Choose the correct alternatives :

(i) The value of cosec(iz) = ______ 1

(a) cosech z

(b) i cosech z

(c) –i cosech z

(d) cosec z

(ii) If the rank of a square matrix is equal to its order then the matrix is : 1

(a) Singular

(b) Non singular

(c) Symmetric

(d) Hermitian

(iii) The value of $e^{-\pi i}$ is : 1

(a) 1

(b) –1

(c) i

(d) –i

(iv) The real part of sin (x + iy) is : 1

(a) sinx coshy

(b) cosx sinhy

(c) sinx sinhy

(d) cosx coshy

(v) If G is finite group of n elements, then order of group G is : 1

(a) n

(b) –n

(c) n + 1

(d) n – 1

(vi) The eigen values of a Hermitian matrix are : 1

(a) all real

(b) pure imaginary

(c) zeros

(d) None of these

(vii) The nature of the roots of an equation of degree n is found by : 1

(a) Cardan's method

(b) Ferraari's method

(c) Descarte's rule of sign

(d) None of these

(e) If a, b, c are integers such that $a \equiv b \pmod m$ then prove that $a + c \equiv (b + c) \pmod m$. 3

(f) If (a, b) = 1 and a | c, b | c then prove that ab | c. 3

UNIT—V

10. (a) Prove that a non-empty subset H of group G is subgroup of G iff :

(i) $a, b \in H \Rightarrow a \cdot b \in H$ and

(ii) $a \in H \Rightarrow a^{-1} \in H$. 4

(b) In a group G, prove that $(ab)^{-1} = b^{-1} \cdot a^{-1}$ for all $a, b \in G$. 3

(c) Prove that G = {1, –1, i, –i} is a cyclic group with respect to multiplication. 3

11. (d) Prove that intersection of any two subgroups of group G is a subgroup of G. 4

(e) Prove that every cyclic group is abelian. 3

(f) In a group G, prove that :

$(ab)^2 = a^2b^2$ for all $a, b \in G$. 3

(e) Define Hermitian matrix and show that :

$$\text{matrix } A = \begin{bmatrix} 2 & 4-i & 6i \\ 4+i & 1 & 3 \\ -6i & 3 & 0 \end{bmatrix}$$

is Hermitian matrix. 3

(f) Prove that every invertible matrix has unique inverse. 3

UNIT—IV

8. (a) Prove that a, b, c $\in$ Z :

(i) $a \mid b \Rightarrow a \mid bc$

(ii) $a \mid b$ and $b \mid c \Rightarrow a \mid c$

(iii) $a \mid b$ and $a \mid c \Rightarrow a \mid (b \pm c)$

(iv) $a \mid b$ and $a \mid c \Rightarrow a \mid (bx \pm cy)$; $\forall$ x, y $\in$ Z. 4

(b) State well order principle and using it prove that there is no integer between 0 and 1. 3

(c) Define equivalence relation and show that on :

A = {3, 1, –5}; R = {3, –5), (–5, 3), (1, 1), (3, 3), (3, 1), (–5, 1), (–5, –5), (1, 3), (1, –5)};

is an equivalence relation. 3

9. (d) Find g.c.d. of 214 and 192 and express it in two ways in the form 214 m + 192 n. 4

(viii) Let R be relation on a non-empty set A then R is reflexive relation if : 1

(a) aRa

(b) aRb $\Rightarrow$ bRa

(c) aRb & bRc $\Rightarrow$ aRc

(d) None of these

(ix) If i and 1 + i are the roots of an equation $x^4 - 2x^3 + 3x^2 - 2x + 2 = 0$ then other roots are : 1

(a) –i, 1 + i

(b) i, 1 – i

(c) –i, –1 – i

(d) –i, 1 – i

(x) The diagonal elements of a Skew-Hermitian matrix are : 1

(a) all zero

(b) all reals

(c) zero or imaginary

(d) all 1's

UNIT—I

2. (a) State and prove Demovier's Theorem for positive integers. 4

(b) If $u=\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)$ then prove that :

$$\tanh\left(\frac{u}{2}\right)=\tan\left(\frac{\alpha}{2}\right).$$ 3

(c) Separate real and imaginary parts of $\tan(x + iy)$. 3

3. (d) Prove that :

$$\tanh^{-1} x=\frac{1}{2}\log\left(\frac{1+x}{1-x}\right);\ |x|<1.$$ 4

(e) If $a = \cos\alpha + i\sin\alpha$ and $b = \cos\beta + i\sin\beta$, then show that $a^2 + b^2 = 2ab\cos(\alpha - \beta)$. 3

(f) Find the all possible values of $(-1)^{1/5}$. 3

UNIT—II

4. (a) Solve the equation $4x^3 + 20x^2 - 23x + 6 = 0$ two of its roots being equal. 4

(b) If f(x) is a polynomial over a field F and $a \in F$, then prove that $(x - a)$ divides f(x) if and only if $f(a) = 0$. 3

(c) Find the condition that the roots of the equation $x^3 - ax^2 + bx - c = 0$ are in A.P. 3

5. (d) Solve the equation $x^3 - 15x = 126$ by Cardan's method. 5

(e) State and prove the remainder theorem for polynomials. 5

UNIT—III

6. (a) Show that the matrix A satisfies its own characteristic equation, where :

$$A=\begin{bmatrix}1 & 3 & 7\\ 4 & 2 & 3\\ 1 & 2 & 1\end{bmatrix}.$$ 4

(b) If λ is characteristic root of matrix A then prove that λ^2 is characteristic root of A^2. 3

(c) Find the eigen values of the matrix :

$$A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}.$$ 3

7. (d) Verify Cayley-Hamilton theorem for the matrix :

$$A=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}.$$ 4