(q) Find the equation of the cone whose vertex is the point (1, 0, 1) and whose guiding curve is $z = 0$, $x^2 + y^2 = 4$. 3

(r) Prove that the sphere

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

will intersect the plane $lx + my + nz = p$ if and only if $(ul + vm + wn + p)^2 \le (l^2 + m^2 + n^2)(u^2 + v^2 + w^2 - d)$. 4

AP–421

B.Sc. (Part–I) Semester–II Examination

MATHEMATICS

(Vector Analysis & Geometry)

Paper—IV

Time—Three Hours] [Maximum Marks—60

Note :— (1) Question No. **1** is compulsory.

(2) Solve **ONE** question from each Unit.

1. Choose the correct alternatives of the following :

(i) If $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar and $x\bar{a} + y\bar{b} + z\bar{c} = 0$ then :

(a) $x + y = z$ (b) $x = y = z = 1$

(c) $x = y = z = 0$ (d) $x = y + z$ 1

(ii) A helix is a twisted curve whose tangent makes a constant angle with a :

(a) Tangent (b) Normal

(c) Fixed direction (d) Binormal 1

(iii) Two non-parallel planes intersect in a :

(a) Line (b) Plane

(c) Point (d) Circle 1

(iv) A surface generated by variable straight line which passes through fixed point and intersects to given curve or touches to given surface is :

(a) Cylinder (b) Sphere

(c) Ellipsoid (d) Cone 1

(v) The point P divides the join of two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ in the ratio K : 1 then P divides P_1P_2 internally if :

(a) $K = 0$ (b) $K > 0$

(c) $K < 0$ (d) None of the above 1

(vi) Which of the following equations does not represent a straight line ?

(a) $x - y + z + 1 = 0,\ x - 3z = 0$

(b) $x = 0,\ y = 0$

(c) $x + y = 0,\ z = 2$

(d) $x + y + 3 = 0,\ 2x + 2y = 7$ 1

(vii) The curve of intersection of two spheres is :

(a) Plane (b) Circle

(c) Sphere (d) Cone 1

(q) Find the equation of the line through (x_1, y_1, z_1) and having the direction cosines l, m, n. 3

(r) Show that the line $\frac{x - x_1}{\ell} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ lies in a plane $ax + by + cz + d = 0$ if $a\ell + bm + cn = 0$ and $ax_1 + by_1 + cz_1 + d = 0$. 3

UNIT–V

10. (a) Find the coordinates of the centre and the radius of the circle $x + 2y + 2z = 15$; $x^2 + y^2 + z^2 - 2y - 4z = 11$. 3

(b) The two spheres

$$x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$$

$$x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$$

will be orthogonal if $2u_1u_2 + 2v_1v_2 + 2w_1w_2 = d_1 + d_2$. Prove this. 3

(c) Find the equation of right circular cylinder of radius 2 and whose axis is the line $\frac{x - 1}{2} = \frac{y}{3} = \frac{z - 3}{1}$. 4

11. (p) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point (1, 2, 3). 3

7. (p) Find the constant a, b, c so that $\bar{f} = (x+2y+az)\bar{i} + (bx-3y-z)\bar{j} + (4x+cy+2z)\bar{k}$ is irrotational. 3

(q) Evaluate $\int_C \bar{F} \cdot d\bar{r}$ from (0, 0, 0) to (1, 1, 1) along the straight line joining (0, 0, 0) to (1, 1, 1) when $\bar{F} = (3x^2 + 6y)\bar{i} - 14yz\bar{j} + 20xz^2\bar{k}$. 4

(r) If $\phi = x^3 + y^3 + z^3 - 3xyz$, find div grad ϕ and curl grad ϕ. 3

UNIT–IV

8. (a) Prove that if p is the length of a perpendicular OA from the origin O to the plane, then its equation can be expressed as lx + my + nz = p, where l, m, n are the direction cosines of OA. 4

(b) Find the equation of the line through the point (1, 2, 3) parallel to the line x − y + 2z = 5, 3x + y + z = 6. 3

(c) Prove that the distance of the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane x + y + z = 2 from the point (3, 4, 5) is 6. 3

9. (p) Find the equation of the plane through the intersection of the planes x + 3y + 6 = 0 and 3x − y − 4z = 0 whose perpendicular distance from the origin is unity. 4

(viii) If $\bar{a} = t\bar{i} - 3\bar{j} + 2t\bar{k}$, $\bar{b} = \bar{i} - 2\bar{j} + 2\bar{k}$ and $\bar{c} = 3\bar{i} + t\bar{j} - \bar{k}$, then $\int_1^2 \bar{a} \cdot (\bar{b} \times \bar{c})\, dt =$.

(a) 0 (b) 2

(c) 3 (d) 5 1

(ix) If $\bar{r} = \bar{r}(t)$ is equation of space curve then curvature K is equal to :

(a) $\frac{[\dot{\bar{r}}\ \ddot{\bar{r}}\ \dddot{\bar{r}}]}{|\dot{\bar{r}} \times \ddot{\bar{r}}|^2}$ (b) $\dot{\bar{r}}/|\dot{\bar{r}}|$

(c) $\frac{\dot{\bar{r}} \times \ddot{\bar{r}}}{|\dot{\bar{r}} \times \ddot{\bar{r}}|}$ (d) $\frac{|\dot{\bar{r}} \times \ddot{\bar{r}}|}{|\dot{\bar{r}}|^3}$ 1

(x) A region in which, a straight line, parallel to coordinate axes, cuts the curve C in more than two points that region is :

(a) Special region

(b) General region

(c) Multiply connected

(d) None of the above 1

UNIT–I

2. (a) Prove that $(\bar{a} \times \bar{b}) \cdot (\bar{c} \times \bar{d}) = \begin{vmatrix} \bar{a}\cdot\bar{c} & \bar{a}\cdot\bar{d} \\ \bar{b}\cdot\bar{c} & \bar{b}\cdot\bar{d} \end{vmatrix}$. 3

(b) If $\bar{r}(t) = 5t^2\bar{i} + t\bar{j} - t^3\bar{k}$, prove that :

$$\int_1^2 \bar{r} \times \frac{d^2\bar{r}}{dt^2}\,dt = -14\bar{i} + 75\bar{j} - 15\bar{k}.$$ 3

(c) If $\bar{r} = a\cos t\,\bar{i} + a\sin t\,\bar{j} + a\,t\tan\alpha\,\bar{k}$, find $|\dot{\bar{r}} \times \ddot{\bar{r}}|$ and $[\dot{\bar{r}}\ \ddot{\bar{r}}\ \dddot{\bar{r}}]$. 4

3. (p) If $\bar{e}$ is the unit vector making an angle θ with x-axis, show that $\frac{d\bar{e}}{d\theta}$ is a unit vector obtained by rotating $\bar{e}$ through a right angle in θ - increasing direction. 3

(q) Given that $\bar{r}(t) = \begin{cases} 2\bar{i} - \bar{j} + 2\bar{k}, & \text{for } t = 2 \\ 4\bar{i} - 2\bar{j} + 3\bar{k}, & \text{for } t = 3 \end{cases}$,

show that $\int_2^3 \bar{r} \cdot \frac{d\bar{r}}{dt}\,dt = 10$. 3

(r) Show that the necessary and sufficient condition for $\bar{f}(t)$ to have constant direction is $\bar{f} \times \frac{d\bar{f}}{dt} = \bar{0}$. 4

UNIT–II

4. (a) If the tangent and the binormal at a point of a curve make angle θ, ϕ respectively with a fixed direction, show that $\frac{\sin\theta}{\sin\phi}\frac{d\theta}{d\phi} = -\frac{\kappa}{\tau}$. 4

(b) For the curve $x = 3t$, $y = 3t^2$, $z = 2t^3$ at the point $t = 1$, find K and $\mathcal{T}$. 4

(c) Prove that for any curve $\bar{t}' \cdot \bar{b}' = -\kappa\tau$. 2

5. (p) Show that Serret-Frenet formulae at a point can be written in the form $\bar{t}' = \bar{d} \times \bar{t}$, $\bar{n}' = \bar{d} \times \bar{n}$, $\bar{b}' = \bar{d} \times \bar{b}$ where $\bar{d} = \tau\bar{t} + \kappa\bar{b}$ is a Darboux's vector. 3

(q) Define three fundamental planes. 3

(r) Prove that the position vector of the current point on a curve satisfies the differential equation

$$\frac{d}{ds}\left(\sigma\frac{d}{ds}(\rho\bar{r}'')\right) + \frac{d}{ds}\left(\frac{\sigma}{\rho}\bar{r}'\right) + \frac{\rho}{\sigma}\bar{r}'' = \bar{0}.$$ 4

UNIT–III

6. (a) If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, find div $\bar{r}$ and curl $\bar{r}$. 3

(b) Find the total work done in moving a particle in a force field given by $\bar{F} = 2xy\bar{i} + 3z\bar{j} - 6x\bar{k}$ along the curve $x = t^2 + 1$, $y = t$, $z = t^3$ from $t = 0$ to $t = 1$. 3

(c) Verify Green's theorem in the plane for

$$\int_C (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy,$$

where C is the boundary of the region R bounded by $y = \sqrt{x}$, $y = x^2$. 4