(f) Solve

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = x\cos(\log x).$$ 4

UNIT – V

10. (a) Solve the D. E.

$(x-1)y''-(2x+1)y'+(x+2)y=(x^2-2x+1)e^{2x}$. 5

(b) Solve the D. E.

$y''+\tan x\, y'+y\cos^2 x=0$ by putting $z=\sin x$. 5

11. (c) Solve the D. E.

$$(x\sin x+\cos x)\frac{d^2y}{dx^2} - x\cos x\frac{dy}{dx}+y\cos x=0.$$ 5

(d) Solve the Simultaneous differential equations

$D^2x - 2y = 0$; $D^2y + 2x = 0$. 5

AR – 508

Second Semester B. Sc. (Part – I) Examination

(Old)

MATHEMATICS – III

Paper – III

(Integration and Differential Equations)

P. Pages : 8

Time : Three Hours] [Max. Marks : 60

Note : (1) Question no. **1** is compulsory.
(2) Attempt **One** question from each unit.

1. Choose the correct alternatives :—

(i) The D.E. $\frac{dy}{dx}+Py=Q\cdot y^n$ is called _____.

(a) Linear D.E. of first order,

(b) Bernouli's Equation,

(c) Exact D.E.,

(d) Homogeneous D.E. 1

(ii) $\int \sin hx\, dx$ is equal to _____.

(a) $\cos hx$,

(b) $-\cos hx$,

(c) $\sin x$,

(d) $\cos x$. 1

(iii) The P.I. of D.E. $f(D)y = e^{ax}$ is equal to ___.

(a) $\frac{1}{f(a)} e^{ax}$,

(b) $\frac{1}{f(-a)} e^{ax}$.

(c) $\frac{1}{f(D+a)} e^{ax}$,

(d) None of these. 1

(iv) $\int \frac{1}{1+x^2} dx$ is equal to _____.

(a) $\frac{1}{2} \tan^{-1} x + c$,

(b) $\tan^{-1} x + c$,

(c) $\cot^{-1} x + c$,

(d) $\sin^{-1} x + c$. 1

(v) The area enclosed by the curve $y = f(x)$, x–axis and the ordinates $x = a$ and $x = b$ is _____.

(a) $\int_a^b y^2 dx$,

(b) Solve $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$. 3

(c) Find the orthogonal trajectories of the family of semicubical parabolas $ay^2 = x^3$. 3

7. (d) Solve $\frac{dy}{dx} = e^{x-y} + 3x^2 e^{-y}$. 3

(e) Solve $y' + y = \frac{1}{1 + e^{2x}}$. 3

(f) Solve $xy - \frac{dy}{dx} = y^3 e^{-x^2}$. 4

UNIT – IV

8. (a) Solve $(D^2 + 9)y = 0$. 3

(b) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$. 3

(c) Solve $\frac{d^2y}{dx^2} + a^2y = \sin ax$. 4

9. (d) Solve $2y'' + 2y' + 3y = 0$. 3

(e) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{-5x}$. 3

UNIT – II

4. (a) Find the Perimeter of the Cardioid
$r = a(1 + \cos\theta)$. 3

(b) Find the area of a circle $x^2 + y^2 = a^2$. 3

(c) Find the volume of the solid generated by the revolution of the loop of the curve
$x = t^2, \quad y = t - \frac{t^3}{3}$. 4

5. (d) Find the length of the arc of the curve
$y = \ln \frac{e^x - 1}{e^x + 1}$ from $x = 1$ to $x = 2$. 3

(e) Find the area of the Cardioid
$r = a(1 + \cos\theta)$. 3

(f) Show that the volume generated by revolving the loop of the curve
$y^2(a + x) = x^2(3a - x)$ about the x – axis is $\pi a^3(8\ln 2 - 3)$. 4

UNIT – III

6. (a) Show that the D. E.
$(\sin x \cdot \sin y - x e^y)\,dy = (e^y + \cos x \cdot \cos y)\,dx$
is exact and hence solve it. 2+2

(b) $\int_a^b x\,dy$,

(c) $\int_a^b y\,dx$,

(d) $\int_a^b x^2\,dy$. 1

(vi) The solution of the D. E. $\frac{dy}{dx} = \frac{x+y}{x-y}$ is obtained by the substitution :

(a) $y = \frac{v}{x}$,

(b) $y = vx$,

(c) $x = \frac{v}{y}$,

(d) None of these. 1

(vii) The volume of a sphere of radius a is _____.

(a) $\frac{2}{3}\pi a^3$,

(b) $\frac{4}{3}\pi a^2 h$,

(c) $\frac{4}{3}\pi a^3$,

(d) $\frac{1}{3}\pi a^3$,

(viii) The Particular integral of $y'' + Py' + Qy = R$ where P, Q, and R are functions of x is $y = e^x$ if _____.

(a) $1 - P + Q = 0$,

(b) $1 + P - Q = 0$,

(c) $1 + P + Q = 0$,

(d) $1 - P = Q$. 1

(ix) The roots of the D.E. $\frac{d^2y}{dx^2} + 4y = 0$ are _____.

(a) Real and equal,

(b) Real and distinct,

(c) Complex,

(d) Surd. 1

(x) If $y_1(x)$ and $y_2(x)$ be any two differentiable functions then the quantity Wronskian $W(y_1, y_2, x)$ is equal to _____.

(a) $\begin{vmatrix} y_1^1(x) & y_2^1(x) \\ y_1(x) & y_2(x) \end{vmatrix}$

(b) $\begin{vmatrix} y_1(x) & y_1^1(x) \\ y_2(x) & y_2^1(x) \end{vmatrix}$

(c) $\begin{vmatrix} y_1(x) & y_2^1(x) \\ y_2(x) & y_1^1(x) \end{vmatrix}$

(d) None of these. 1

UNIT – I

2. (a) Evaluate $\int_0^1 x \cdot (1-x)^{\frac{1}{2}}\, dx$. 3

(b) Evaluate $\int_0^1 \frac{1 - 4x + 2x^2}{\sqrt{2x - x^2}}\, dx$. 3

(c) Prove that

$$\int \cot^n x \cdot dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x \cdot dx$$

and hence find $\int \cot^4 x \cdot dx$. 3 + 1

3. (d) Evaluate $\int \frac{\cos x}{\sin^2 x + 3 \sin x + 2}\, dx$. 3

(e) Evaluate $\int (2x + 5)\sqrt{x^2 + 3x + 1}\, dx$. 3

(f) Prove that

$$\int \sec^n x \cdot dx = \frac{\sec^{n-2} x}{n-1} \cdot \tan x + \left(\frac{n-2}{n-1}\right) \int \sec^{n-2} x \cdot dx$$

and hence find $\int \sec^3 x\, dx$. 3 + 1