AP–442

B.Sc. (Part–II) Semester—III Examination
MATHEMATICS—V
(Advanced Calculus)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory.

(2) Attempt **ONE** question from each unit.

1. Choose correct alternative :

(i) Let $< s_n >$ be a sequence such that $\lim_{n\to\infty} s_n = \ell$ and $s_n \geq 0, \forall n \in N$ then :

(a) $\ell < 0$ (b) $\ell \geq 0$

(c) $\ell = -2$ (d) $\ell = -1$ 1

(ii) If $< s_n >, < t_n >$ and $< u_n >$ be three sequences such that $s_n \leq t_n \leq u_n, \forall n \in N$ and $\lim_{n\to\infty} s_n = \lim_{n\to\infty} u_n = \ell$ then $\lim_{n\to\infty} t_n$ is :

(a) 0 (b) ℓ

(c) $-\ell$ (d) m 1

(iii) Let Σa_n be a series of positive terms such that $\lim_{n\to\infty} n\sqrt{a_n} = \ell$, $a_n \geq 0 \; \forall n$. Then Σa_n is cgt if :

(a) $\ell = 1$ (b) $\ell < 1$

(c) $\ell > 1$ (d) $\ell = 2$ 1

(iv) The series $\sum \frac{1}{n^p}$ is dgt if :

(a) $p > 1$ (b) $p \leq 1$

(c) $p = 2$ (d) $p = 3$ 1

(v) If J is the Jacobian of x and y with respect to u and v and J' is Jacobian of u, v with respect to x and y then J' is :

(a) $\frac{1}{J}$ (b) J

(c) $-J$ (d) J^2 1

(vi) If iterated limits of a function are not equal at point then :

(a) limit exist at point

(b) limit does not exist

(c) limit is zero

(d) None of these 1

(vii) The value of $\left\lceil \tfrac{1}{2} \right.$ is :

(a) 0 (b) 1

(c) π (d) $\sqrt{\pi}$ 1

(viii) The correct value of $\int_0^1 \int_0^3 dxdy$ is :

(a) 0 (b) -3

(c) $+3$ (d) 1 1

11. (a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dy \, dx$ by changing the order of integration. 5

(b) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{\sqrt{x^2+y^2}} dx \, dy$ changing to polar coordinates. 5

UNIT—IV

8. (a) Prove that $\beta(m, n) = \int_0^1 \frac{(x^{m-1} + x^{n-1})}{(1+x)^{m+n}} dx$. 4

(b) Prove that $\beta(m, n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$. 3

(c) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{(1+x^2+y^2)} dx\, dy$. 3

9. (a) Prove that $\int_0^\infty \frac{x^{m-1}}{(ax+b)^{m+n}} dx = \frac{\overline{|m}\ \overline{|n}}{a^m b^n \overline{|m+n}}$. 4

(b) Evaluate $\int_0^\infty \sqrt{x}\, e^{-3\sqrt{x}}\, dx$. 3

(c) Evaluate $\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}} dx$. 3

UNIT—V

10. (a) Change the order of integral $\int_0^{2a} \int_{x^2/3}^{3a-x} f(x, y)\, dx\, dy$. 5

(b) Evaluate $\int_0^a \int_y^a \frac{x^2}{(x^2+y^2)^{3/2}} dx\, dy$ by changing into polar coordinates. 5

(ix) The value of $\beta(½, ½)$ is :

(a) π (b) $-\pi$

(c) ½ (d) 1 1

(x) The value of $\int_0^\pi \int_0^1 dr\, d\theta$ is :

(a) π (b) π^2

(c) $\frac{1}{\pi}$ (d) 0 1

UNIT—I

2. (a) If $<s_n>$, $<t_n>$ and $<u_n>$ be three sequences such that $s_n \le t_n \le u_n$, $\forall n$ and $\lim_{n\to\infty} s_n = \lim_{n\to\infty} u_n = \ell$ then prove that $\lim_{n\to\infty} t_n = \ell$. 4

(b) Show that sequence :

$<s_n>, s_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}$

is monotonic and bounded. 3

(c) Evaluate $\lim_{n\to\infty} \left(\frac{6+n}{1-2n}\right)^3$. 3

3. (a) Prove that every Cauchy sequence is bounded. 4

(b) Show that sequence $<s_n>$, where $s_n = \frac{1}{1!} + \frac{1}{2!} + \dots \frac{1}{n!}$ is convergent. 3

(c) Evaluate $\lim\limits_{n\to\infty}\frac{1+2+3+...+n}{n^2}$. 3

UNIT—II

4. (a) Prove that the series $\sum x_n$ converges iff for every $\epsilon > 0$ there exist $M \in N$ such that :

$$m \geq n \geq M \Rightarrow |x_{n+1} + x_{n+2} + ... + x_m| < \epsilon . \qquad 4$$

(b) Test the convergence of $\sum \frac{n!}{n^n}$. 3

(c) Test the convergence of series $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+.....$ 3

5. (a) Let $\sum a_n$ be a sequence of real numbers such that : $\ell = \lim\limits_{n\to\infty} n\sqrt{a_n}$, $a_n \geq 0$, $\forall n \in N$. Then prove that $\sum a_n$ is convergent if $\ell < 1$. 4

(b) Test the convergence of $\sum \left(\frac{n}{n+1}\right)^{n^2}$. 3

(c) Test the convergence of series $\sum \frac{1}{n^2+1}$ by using integral test. 3

UNIT—III

6. (a) If x, y are differentiable functions of u, v and u, v are differentiable functions of r, s then prove that :

$$\frac{\partial(x,y)}{\partial(u,v)} \times \frac{\partial(u,v)}{\partial(r,s)} = \frac{\partial(x,y)}{\partial(r,s)} \qquad 4$$

(b) Expand $x^3 + y^3 - 3xy$ at point (2, 3). 3

(c) Prove that $\lim\limits_{(x,y)\to(1,1)}(x^2+2y) = 3$ by ϵ-δ definition. 3

7. (a) Let f(x, y) be defined in an open region D and it has a local maximum or local minimum at (x_0, y_0). If partial derivatives f_x and f_y exist at (x_0, y_0). Then prove that $f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) = 0$. 4

(b) Using ϵ-δ definition, prove that $\lim\limits_{(x,y)\to(2,1)}(xy) = 2$. 3

(c) Show that for the function $f(x, y) = \frac{xy}{x^2+y^2}$ limit does not exist at (0, 0) even though iterated limits are equal. 3