AP–443

B.Sc. (Part–II) Semester—III Examination

MATHEMATICS—VI

(Partial Differential Equations)

Time : Three Hours] [Maximum Marks : 60

Note :— Question No. **1** is compulsory and solve **ONE** question from each unit.

1. Choose the correct altenatives : 10

(i) The PDE of the relation $z = (x + a)(y + b)$, where a and b are arbitray constants is ______.

(a) $p = q$ (b) $z = pq$

(c) $px = qy$ (d) None of these

(ii) Lagrange's form of the partial differential equation of order one is ______.

(a) $Pp - Qq = R$

(b) $Pp + Qq = R$

(c) $Pq + Qp = R$

(d) None of these

(iii) The value of $\frac{1}{2D-3D'}e^{x-y}$ is ________.

(a) e^{x-y} (b) $\frac{1}{5}e^{x-y}$

(c) $-e^{x-y}$ (d) $-\frac{1}{5}e^{x-y}$

(iv) The solution of PDE $(D - mD')Z = 0$ has the form :

(a) $Z = F(y + mx)$

(b) $Z = F(y - mx)$

(c) $Z = F(x - my)$

(d) None of these.

(v) The diffusion equation $z_{xx} = z_t$ is ________ .

(a) Elliptic

(b) Parabolic

(c) Hyperbolic

(d) None of these.

11. (a) Use the method of separation of variable to solve the equation $\frac{\partial^2 v}{\partial x^2} = \frac{\partial v}{\partial t}$, given that $v = 0$ when $t \to \infty$ as well as $v = 0$ at $x = 0$ and $x = l$. 5

(b) Obtain the partial differential equation of vibrating string. 5

UNIT—IV

8. (a) Define the nth order distance. Find the distance between the curve $y(x) = xe^{-x}$, $y_1(x) = 0$ on $[0, 2]$. 1+4

(b) Show that the curves $y(x) = \frac{\cos n^2 x}{n}$ and $y_1(x) = 0$, $x \in [0, \pi]$ are close in the sense of zero order proximity but not of the first order proximity. 5

9. (a) Define linear functional. Show that the functional : $L[y(x)] = \int_a^b [2y(x) - 3y'(x)]\,dx$ defined on 'M' is linear. 1+4

(b) Define increment in the functional. Find δI for $I[y] = \int_0^{\pi} y' \sin y \, dx$. 1+4

UNIT—V

10. (a) Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that when $x = 0$, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$. 5

(b) Solve by Separation Method :

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0.$$ 5

(vi) The PDE of the form $Rr + Ss + Tt + f(x, y, z, p, q) = 0$ is parabolic at the point (x, y) if ______.

(a) $S^2 - 4RT > 0$

(b) $S^2 - 4RT < 0$

(c) $S^2 - 4RT = 0$

(d) None of these

(vii) The founder of calculus of variations is ______.

(a) Lagrange

(b) Euler

(c) J. Bernoulli

(d) Leibnitz

(viii) A variable quantity whose value is determined by one or more than one function is called ______.

(a) An extremum

(b) A point of inflection

(c) A functional

(d) None of these.

(ix) The PDE $\nabla^2 u = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ is ________.

(a) Wave equation

(b) Laplace equation

(c) Diffusion equation

(d) None of these.

(x) Which of the following PDE is the equation of the vibrating string ?

(a) $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ (b) $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$

(c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial t}$ (d) None of these

UNIT—I

2. (a) Apply Charpit method to solve $z^2 = pqxy$. 4

(b) Solve $p^2 + q^2 = k^2$. 3

(c) Find the general solution of the PDE $x^2p + y^2q = (x + y)z$. 3

3. (a) Solve the PDE $x(y^2 - z^2)\, p + y\,(z^2 - x^2)q = z(x^2 - y^2)$. 4

(b) Solve $p + q = pq$ by Charpit Method. 3

(c) Form a partial D.E. of the relation :

$$f(x + y + z,\ x^2 + y^2 + z^2) = 0$$

by eliminating arbitrary function. 3

UNIT—II

4. (a) Solve $(D^2 - DD')\, z = \cos x \cos 2y$. 5

(b) Solve $(D^3 - 6D^2D' + 11DD'^2 - 6D'^3)\, z = 0$. 3

(c) Find particular integral of $(D^2 - D')z = 2y - x^2$. 2

5. (a) Solve $r + s - 6t = y \cos x$. 5

(b) Solve $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x + 2y)$. 5

UNIT—III

6. (a) Reduce the equation $r = x^2t$ to canonical form. 5

(b) Solve $r - a^2t = 0$ by Monge's Method. 5

7. (a) Solve $r + 3s + t + (rt - s^2) = 1$ by Monge's Method. 5

(b) Reduce the Tricomi equation $Z_{xx} + xZ_{yy} = 0,\ x \neq 0$ to canonical form. 5