B.Sc. Part—II Semester—IV Examination

4S : MATHEMATICS (Old)

(Mechanics)

Paper—VIII

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory and attempt it at once only.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternative in the following :

(i) Two parallel forces not having the same line of action form a couple if they are :

(a) Like and equal (b) Unlike and equal

(c) Unlike and unequal (d) Like and unequal 1

(ii) If the displacement is measured in the direction of force, then the work done is :

(a) Positive (b) Negative

(c) Constant (d) Zero 1

(iii) The line of symmetry of catenary is called as :

(a) The directrix of catenary (b) The span of catenary

(c) The sag of catenary (d) The axis of catenary 1

(iv) The Cartesian equation of uniform catenary is :

(a) $s = c \tan \psi$ (b) $c = s \tan \phi$

(c) $y = c \cosh x/c$ (d) $y = c \sinh(x/c)$ 1

(v) The normal component of acceleration is :

(a) $\frac{v}{\rho}$ (b) $\frac{d^2\rho}{dt^2}$

(c) $\frac{v^2}{\rho}$ (d) $\frac{d\rho}{dt}$ 1

(vi) The greatest height attained by the projectile is :

(a) $\frac{u^2 \sin^2 \alpha}{2g}$ (b) $\frac{u \sin^2 \alpha}{g}$

(c) $\frac{u \sin \alpha}{g}$ (d) $\frac{u^2 \sin^2 2\alpha}{g}$ 1

(vii) A simple pendulum with variable length has :

(a) Zero degree for freedom (b) One degree of freedom

(c) Two degrees of freedom (d) Three degrees of freedom 1

(viii) If q_j are generalised co-ordinates and constraints are holonomic, then δq_j are :

(a) Independent (b) Dependent

(c) Similar (d) None of these 1

(ix) In a central force field, the angular momentum of particle :

(a) Is variable (b) Depends on force

(c) Remains constant (d) None of these 1

(x) The radius vector drawn from the planet to the sun sweeps out equal areas in equal times is the statement of :

(a) The inverse square law (b) Laws of motion

(c) Conservation law of areal velocity (d) Kepler's law 1

UNIT—I

2. (a) If forces $P_1, P_2, P_3, P_4, P_5, P_6$ act along the sides of a regular hexagon taken in order, then prove that the forces will be in equilibrium if :

$P_1 + P_2 + P_3 + P_4 + \ldots + P_6 = 0$ and

$P_1 - P_4 = P_3 - P_6 = P_5 - P_2$. 5

(b) State and prove Lami's Theorem. 5

3. (p) Prove that any system of forces acting at different points of a rigid body can be reduced to a single force through a given point and a couple. 5

(q) If the moments of a system of forces (not in equilibrium) acting on a rigid body in one plane about three colinear points A, B, C in the plane are G_1, G_2, G_3 then prove that $G_1 \cdot BC + G_2 \cdot CA + G_3 \cdot AB = 0$. 5

UNIT—II

4. (a) Prove that the necessary and sufficient condition that a rigid body acted upon by a system of coplanar forces be in equilibrium is that sum of virtual work done by the forces in any small displacement consistent with the geometrical conditions of the system is zero. 5

   (b) If five weightless rods of equal length are joined together so as to form rhombus ABCD with one diagonal BD. If a weight W be attached to G and system is suspended from A. Then show that there is a thrust in BD equal to $\frac{W}{\sqrt{3}}$. 5

5. (p) Find the Cartesian equation of uniform catenary and also find relation between s and x. 5

   (q) If a uniform chain of length 2ℓ has its end fixed at two points in the same level and if the sag at the middle is h, then prove that the span is $\frac{\ell^2 - h^2}{h} \log\left(\frac{\ell + h}{\ell - h}\right)$. 5

UNIT—III

6. (a) Find radial and transverse components of velocity and acceleration of a particle. 5

   (b) If a particle moves in a catenary and the direction of its acceleration at any point makes equal angles with the tangent and normal to the path at that point and if the speed at the vertex, where $\psi = 0$ be u, then show that velocity and acceleration at any other point are given by ue^{ψ} and $\left(\sqrt{2}/c\right)u^2 e^{2\psi}\cos^2\psi$. 5

7. (p) Define time of flight and horizontal range of projectile and find the equation to the path of projectile. 5

   (q) A projectile started from O at an elevation α. After t seconds, its position appeared to have an elevation β as seen from O. Prove that its initial velocity was $\frac{gt\cos\beta}{2u\sin(\alpha - \beta)}$. 5

UNIT—IV

8. (a) State and prove D'Alembert's principle for the system of particles. 5

   (b) Construct the Lagrangian for a particle moving in space and then deduce the equation of motion. 5

9. (p) Derive the Lagrange's equations of motion for conservative system from D'Alembert's principle. 5

(q) Two particles of masses m_1 and m_2 are connected by a light inextensible string which passes over a small smooth fixed pulley. If $m_1 > m_2$, then prove that common acceleration of particle is $\{(m_1 - m_2)/(m_1 + m_2)\}g$. 5

UNIT—V

10. (a) Show that the path of a particle moving under central force is given by :

$$\left(\frac{d^2u}{d\phi^2}\right) + u = -(m/h^2u^2)\, F(1/u), \text{ where } u = \frac{1}{r}. \quad 5$$

(b) Prove that in a central force field the areal velocity is conserved. 5

11. (p) State and prove Virial theorem. 5

(q) A particle moves on curve $r^n = a^n \cos n\theta$ under the influence of a central force field. Find law of force. 5