AQ—695

B.Sc. (Part—II) Semester-IV Examination

4S : MATHEMATICS

(Mechanics)

Paper—VIII

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Question **ONE** is compulsory and attempt it at once only.

(2) Attempt **one** question from each Unit.

1. Choose the correct alternatives :

(i) Two parallel forces not having the same line of action form a couple if they are :

(a) like and equal

(b) unlike and unequal

(c) like and unequal

(d) unlike and equal 1

(ii) The lowest point of the catenary is called its :

(a) vertex

(b) sag

(c) span

(d) directrix 1

(iii) The magnitude of the resultant R of two forces of magnitudes P and Q acting at an angle π is :

(a) $P + Q$

(b) $P - Q$

(c) $\sqrt{P^2 + Q^2}$

(d) $\sqrt{P^2 - Q^2}$ 1

(iv) The intrinsic equation of common catenary is :

(a) $y = c \cosh\left(\frac{x}{c}\right)$

(b) $y = c \sinh\left(\frac{x}{c}\right)$

(c) $s = c \tan \psi$

(d) none of these 1

(v) The tangential component of acceleration is :

(a) $\frac{d^2s}{dt^2}$ (b) $\frac{ds}{dt}$

(c) v/p (d) none of these 1

(vi) The path of the projectile is called :

(a) Angle of projection

(b) Trajectory

(c) Horizontal range of projectile

(d) None of these 1

9. (p) Discuss the motion of a particle in a plane by using polar coordinates. 4

(q) Prove that the Lagrange's equation's of motion can be written in the form :

$$\left(\frac{\partial L}{\partial \dot{q}_i}\right)^{\cdot} - \left(\frac{\partial L}{\partial q_i}\right) = Q_i^1, \quad i = 1, 2, \ldots\ldots, n$$

for a system which is partly conservative. The quantity L refers to the conservative part and Q_i^1 to the forces which are not conservative. 6

UNIT—V

10. (a) Prove that for a central force field F, the path of a particle of mass m is given by :

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{h^2a^2} F\left(\frac{1}{u}\right), \quad u = \frac{1}{\gamma}. \qquad 5$$

(b) Prove that for a particle moving under a central force such that $V = Kr^{n+1}$, the Virial theorem reduces to : $\overline{2T} = (n+1)\overline{V}$. 5

11. (p) State and prove Virial theorem. 5

(q) A particle moves on a curve $r^n = a^n \cos n\theta$ under the influence of a central force field. Find the law of force. 5

(b) Find the expressions for radial and transverse components of accelerations of a particle moving along a plane curve. 5

7. (p) A particle aimed at a mark, which is in the horizontal plane through the point of projection, falls a feet short of it when the elevation is α and goes b feet too far when the elevation is β. Show that if the velocity of projection B in each case be same, then the proper elevation is $\frac{1}{2}\sin^{-1}\left(\frac{a\sin 2\beta + b\sin 2\alpha}{a+b}\right)$. 5

(q) If v and w be the velocities at the end of a focal chord of a projectile's path and u is the horizontal component of velocity. Show that :

$$\frac{1}{v^2} + \frac{1}{w^2} = \frac{1}{u^2}$$ 5

UNIT—IV

8. (a) Show that the shortest distance between two points in a plane is a straight line. 5

(b) State and prove D'Alembert's principle for a system of particles. 5

(vii) The greatest height attained by the projectile is :

(a) $\frac{u^2\sin^2\alpha}{2g}$

(b) $\frac{u\sin^2\alpha}{g}$

(c) $\frac{u\sin\alpha}{g}$

(d) $\frac{u^2\sin^2 2\alpha}{g}$ 1

(viii) The shortest distance between two points in a plane is :

(a) an ellipse

(b) a circle

(c) a parabola

(d) a straight line 1

(ix) The radius vector drawn from the planet to the sun sweeps out equal areas in equal times. This statement gives :

(a) the inverse square law

(b) time average

(c) the conservation of areal velocity

(d) none of these 1

(x) In a central force field, the angular momentum of a particle remains :

(a) zero

(b) constant

(c) positive

(d) none of these 1

UNIT—I

2. (a) Three forces P, Q, R act along the sides of the triangle formed by the lines $x + y = 1$, $y - x = 1$, $y = 2$. Find the equation of the line of action of their resultant. 5

(b) Prove that any system of coplanar forces acting at different points of a rigid body can be reduced to a single force through a given point and a single couple. 5

3. (p) State and prove Lami's theorem. 5

(q) A heavy uniform rod AB of weight W is hinged at A to a fixed point and rests in a position inclined at angle α to the horizontal. It is acted upon by a horizontal force P applied at the lower end B. Find the action at the hinge and the magnitude of P. 5

UNIT—II

4. (a) Derive the Cartesian equation of common catenary. 5

(b) Define span and if a uniform chain, of length ℓ is to be suspended from two points A and B in the same horizontal line so that either terminal tension is n times that at the lowest point then show that its span AB is $\frac{1}{\sqrt{n^2-1}}\ell n\left[n+\sqrt{n^2-1}\right]$ 5

5. (p) Four equal rods of length 2a and weight W are smoothly joined to form a rhombus ABCD, which is kept in shape by a light rod BD. Angle BAD is 60° and the rhombus is suspended in a vertical plane from A. Find the thrust in BD. 5

(q) Show that the necessary and sufficient condition that a rigid body acted upon by a number of coplanar forces at different points is in equilibrium, is that the algebraic sum of the virtual work done by the forces in any small arbitrary virtual displacement consistent with the geometrical conditions of the system is zero. 5

UNIT—III

6. (a) The velocities of a particle along and perpendicular to the radius vector from a fixed origin are λr^2 and $\mu\theta^2$. Show that the equation to the path is $\frac{\lambda}{\theta} = \frac{\mu}{2r^2} + c$ and the components of acceleration are $2\lambda^2 r^3 - \frac{\mu^2\theta^4}{r}$ and $\lambda\mu r\theta^2 + 2\frac{\mu^2\theta^3}{r}$. 5