AS–1425

B.Sc. Part—II Semester—IV Examination

MATHEMATICS (Old)

(Mechanics) Paper—VIII

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory and attempt it at once only

(2) Attempt **one** question from each unit.

1. Choose the correct alternative in the following :

(i) The lowest point of catenary is called its _____ . 1

(a) vertex (b) sag

(c) span (d) directrix

(ii) The Magnitude of the resultant $\overline{R}$ of two forces of magnitude $\overline{P}$ and $\overline{Q}$ acting at an angle 90° is _____ . 1

(a) $\overline{P}+\overline{Q}$ (b) $\overline{P}-\overline{Q}$

(c) $\sqrt{P^2+Q^2}$ (d) $\sqrt{P^2-Q^2}$

(iii) If a system of forces acts upon a body such that the body remains at rest, then the forces are said to be _____ . 1

(a) in equilibrium (b) a couple

(c) a single force (d) resultant

(iv) The virtual work on a mechanical system by applied forces and reverse effective forces is _____ . 1

(a) zero (b) equivalent

(c) independent (d) none of these

(v) The intrinsic equation of common catenary is _____ . 1

(a) $y = x \tan c$ (b) $y = c \cosh (x/c)$

(c) $s = c \tan \psi$ (d) none of these

(vi) Tangential component of acceleration is _____ . 1

(a) $\frac{d^2s}{dt^2}$ (b) $\frac{ds}{dt}$

(c) v/p (d) none of these

(vii) The number of independent quantities required to specify the position of the system completely is called the _____. 1

(a) degree of equation (b) degree of path

(c) degrees of freedom (d) none of these

(viii) The path of projectile is known as _____ . 1

(a) Angle of projection (b) Greatest height

(c) Span (d) Trajectory

(ix) 'The radius vector drawn from the planet to the sun sweeps out equal areas in equal times' is known as _____ . 1

(a) Time average (b) Areal velocity

(c) The inverse square law (d) Kepler's law

(x) In a central force field, the angular momentum of a particle remains _____. 1

(a) zero (b) positive

(c) negative (d) constant

UNIT—I

2. (a) State and prove Lami's theorem. 1+4

(b) Prove that the system of coplanar forces acting at a different points of a rigid body can be reduced to a single force through a given point and a single couple. 5

3. (p) If the moments of a system of forces (not in equilibrium) acting on a rigid body in one plane about three colinear points A,B,C in the plane are G_1, G_2, G_3 then show that $G_1.BC + G_2.CA + G_3.AB = 0$. 5

(q) Three forces $\overline{P}, \overline{Q}, \overline{R}$ act along the sides of the triangle formed by the lines $x + y = 1$, $y - x = 1$, $y = 2$, find the equation of the line of action of their resultant. 5

UNIT—II

4. (a) Find the intrinsic equation of a uniform catenary. 5

   (b) Define the span and show that if a uniform chain, of length 'l' is to be suspended from two points 'A' & 'B' in the same horizontal line so that either terminal tension is n times that at the lowest point, then its span AB is $\frac{\ell}{\sqrt{n^2-1}}\log\left[n+\sqrt{n^2-1}\right]$ 5

5. (p) Prove that the necessary and sufficient condition that a particle acted upon by a system of coplanar forces be in equilibrium is that the sum of virtual work done by the forces in any small displacement consistent with the geometrical condition of the system is zero. 5

   (q) Five weightless rods of equal length are joined together so as to form rhombus ABCD with one diagonal BD. If a weight W be attached to C and the system be suspended from A, then show that there is a thrust in BD equal to $W/\sqrt{3}$. 5

UNIT—III

6. (a) Find the tangential and normal components of velocity and acceleration. 5

   (b) A particle moves along a catenary $S = c \tan \psi$ and direction of its acceleration at any point makes equal angles with the tangent and normal to the path at that point. If the speed at the vertex, where $\psi = 0$ be u, then show that the velocity and acceleration at any other point are given by ue^{ψ} and $\left(\sqrt{2}/c\right)u^2e^{2\psi}\cos^2\psi$. 5

7. (p) If a particle is projected at an angle α to the horizontal moving under gravity, then find its horizontal range, time of flight and maximum horizontal range. 5

   (q) Find the equation of path of projectile and also find the expressions for Greatest height of a particle projected in vertically upward with angle α and initial velocity 'u' 5

UNIT—IV

8. (a) Show that the shortest distance between two points in a plane is a straight line. 5

   (b) State and prove D' Alembert's Principle for a system of particles. 5

9. (p) Show that for a system which is partly conservative, the Lagrange's equation of motion can be written in the form $\frac{d}{dt}\left[\frac{dL}{dq_j}\right]-\left[\frac{dL}{dq_j}\right]=Q'_j$ where L refers to the conservative part and Q'_j to the forces which are non-conservative. 5

(q) Construct a Lagrangian for a Spherical pendulum and then obtain the Lagrange's equations of motion. 5

UNIT—V

10. (a) Prove that in a central force field the areal velocity is conserved. 4

(b) State and prove the virial theorem. 6

11. (p) Prove that for a central force field f, the path of a particle of mass 'm' is given by $\frac{d^2u}{d\theta^2}+u=-\frac{m}{h^2u^2}F\left(\frac{1}{u}\right)$, where $u=1/r$. 5

(q) If a particle moves on a circle $r^n = a^n \cos n\theta$ under the influence of a central force field. Find the law of force. 5