11. (p) Prove that for a central force field F, the path of a particle of mass m is given by

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{h^2u^2} F\left(\frac{1}{u}\right), \quad u = 1/\varepsilon. \qquad 5$$

(q) Prove that in a control force field the areal velocity is conserved. 5

AR – 558

Fourth Semester B. Sc. (Part - II) Examination

MATHEMATICS

Paper - VIII

Mechanics

P. Pages : 8

Time : Three Hours] [Max. Marks : 60

Note : (1) Question **one** is compulsory and attempt it once only.

(2) Attempt **One** question from each unit.

1. Choose the correct alternative :—

(i) The sum of moments of a system of coplaner forces about any point is equal to————.

(a) The resultant of the forces about the same point.

(b) The moment of their resultant about the same point.

(c) The moment of coupie about the same point.

(d) The arm of couple at the same point. 1

(ii) The intrinsic equation of comman catenary is ————

(a) $y = c \cos h(n/c)$

(b) $y = c \sin h(n/c)$

(c) $s = c \tan \Psi$

(d) None of these. 1

(iii) The tangential component of acceleration is ————.

(a) $\frac{d^2s}{dt^2}$ (b) $\frac{ds}{dt}$

(c) v/ϱ (d) None of these. 1

(iv) The greatest height attended by the projectile is ————.

(a) $\frac{u^2\sin^2\alpha}{2g}$ (b) $\frac{u\sin^2\alpha}{g}$

(c) $\frac{u\sin\alpha}{g}$ (d) $\frac{u^2\sin^2 2\alpha}{g}$. 1

(v) In a central force field, the angular momentum of a particle remains————.

(a) Zero (b) Constant

(c) Positive (d) None of these. 1

of the line OA to the horizontal, then prove that $t_1^2+2t_1t_2\sin\theta+t_2^2$ is independent of θ. 5

UNIT IV

8. (a) Derive the Lagrange's equations of motion in the form

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0,\ i=1,\ 2\text{---}n$$

for conservative system from D' Alembert's principle. 5

(b) Discuss the motion of a particle in a plane by using polar co–ordinates. 5

9. (p) State and prove D' Alembert principle. 5

(q) Construct a Lagrangian for a spherical pendulum and then obtain the Lagrange's equations of motion. 5

UNIT V

10. (a) State and prove Virial theorem. 5

(b) A particle moves on a curve $\varepsilon^n = a^n \cos n\theta$ under the influence of a central force field. Find the Law of Force. 5

(q) A uniform chain has a horizontal span of 96 ft and the tension at the upper end is twice that at the lowest point. Show that the length of the chain is $\frac{96\sqrt{3}}{\ln(2+\sqrt{3})}$ ft. 5

UNIT III

6. (a) Derive the expressions for radial and transverse velocity and acceleration of a moving particle in a plane curve. 5

(b) A particle moves along a circle $r = 2a\cos\theta$ in such a way that its acceleration towards the origin is always zero, prove that $\frac{dw}{dt} = -2w^2\cot\theta$, where $w = \frac{d\theta}{dt}$. 5

7. (p) A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If α and β be the base angles and θ the angle of projection, prove that $\tan\theta = \tan\alpha + \tan\beta$. 5

(q) If t_1 and t_2 be the times of flight from a point O to a point A and θ be the inclination

(vi) Any system of coplanar forces acting at different points of a rigid body can be reduced to a ————.

(a) Single force

(b) Couple

(c) Single force and couple

(d) Point. 1

(vii) If qi's are the generalized co–ordinates and the constraints are holonomic then δqi's are————.

(a) Dependent (b) Independent

(c) Zero (d) Equivalent. 1

(viii) The square of the periodic time of the planet is proportional to the cube of the—— ————.

(a) Foci of its orbit

(b) Minor axis

(c) Semimajor axis

(d) None of these. 1

(ix) The shortest distance between two points in a space is ————

(a) An ellipse (b) A circle.

(c) A straight line (d) Parabola. 1

(x) The line of symmetry of catenary is called———— of catenary.

(a) Directrix (b) Vertex.

(c) Span (d) Axis. 1

UNIT I

2. (a) Prove that any system of coplanar forces acting at different points of a rigid body can be reduced to a single force or a couple. 5

(b) The algebraic sum of the moments of a system of coplanar forces about points whose co-ordinates are (1, 0), (0, 2) and (2, 3), referred to rectangular axes. are G_1, G_2 and G_3 respectively. Find the tangent of the angle which the direction of the resultant force makes with the axis of x. 5

3. (p) State and prove Lami's theorem. 5

(q) A heavy uniform rod AB of weight W is hinged at A to a fixed point and rests in a position inclined at angle α to the

horizontal. It is acted upon by a horizontal force P applied at the lower end B. Find the action at the hinge and magnitude of P. 5

UNIT II

4. (a) Prove that the necessary and sufficient condition that a particle acted upon by a system of coplanar forces be in equilibrium is that the sum of the virtual work done by the forces in any small displacement consistant with the geometrical conditions of the system is zero. 5

(b) A string of length a, forms the shorter diagonal of a rhombus formed of four uniform rods, each of length b and weight w, which are hinged together. If one of the rods be supported in a horizontal position. Prove that the tension of the string is :

$$\frac{2w(2b^2-a^2)}{b\sqrt{4b^2-a^2}}$$ 5

5. (p) Derive cartesian equation of the uniform catenary. 5