(c) Define :

(i) Field

(ii) Ring with no zero divisor. 1+1

9. (d) Define prime field.

Prove that every prime field of finite characteristic $p > 0$ is isomorphic to the field Z_p. 1+4

(e) Prove that the set of units in a commutative ring with unity is a multiplicative abelian group. 5

UNIT–V

10. (a) State and prove remainder theorem. 1+3

(b) Prove that the polynomial $f(x) = x^4 + 2x + 2$ is irreducible over the field of rational number. 3

(c) If P is a prime number, prove that the polynomial $x^n - p$ is irreducible over the rationals. 3

11. (d) Let $f(x) = a_0 + a_1x + \ldots\ldots + a_nx^n$ be a polynomial with integer coefficients. Suppose that for some prime number P, $P \times a_n$, $P|a_1$, $P|a_2$ .... , $P|a_{n-1}$, $P|a_0$, $P^2 \times a_0$. Then prove that f(x) is irreducible over the rationals. 5

(e) Let R[x] be ring of polynomials over ring R. Then prove that R is a commutative ring if and only if R[x] is a commutative ring. 4

(f) Define. Primitive polynomial. 1

B.Sc. (Part-III) Semester–V Examination

MATHEMATICS–X

(Modern-Algebra)

Time—Three Hours] [Maximum Marks—60

Note:– Question No. 1 is compulsory and solve **one** question from each unit.

1. Choose the correct alternative (1 mark each) :

(i) N is normal subgroup of G iff :

(a) $N_aN_b = N_a + N_b$

(b) $N_aN_b = N_{ab}, \forall\ a,b \in G$

(c) $N_aN_b = N_bN_a$

(d) $N_aN_b = N_a - N_b, \forall\ a,b \in G$

(ii) A group having only improper normal subgroup is called :

(a) a permutation group

(b) a finite group

(c) a simple group

(d) none of these.

(iii) Map $\phi : G \rightarrow G^1$ is an onto homomorphism iff :

(a) G and G^1 are isomorphic

(b) G and G^1 are endomorphic

(c) G and G^1 are homomorphic

(d) None of these.

(iv) If f be a homomorphism of group G onto G^1 with Kernel K, then G^1 is :

(a) Isomorphic to G/K

(b) Isomorphic to K/G

(c) Isomorphic to G

(d) One-one homomorphism.

(v) A field which contains no proper subfield is :

(a) Prime field

(b) Subfield

(c) Division Ring

(d) Integral domain.

(vi) A division ring must contain at least :

(a) One element

(b) Two elements

(c) Three elements

(d) None of these.

5. (c) Show that every homomorphic image of an abelian group is abelian but the converse in not true. 5

(d) Let N be a normal subgroup of G. Define the mapping $f : G \rightarrow G/N$ such that $f(x) = N_x \ \forall \ x \in G$. Then prove that f is a homomorphism of G onto G/N. 5

UNIT–III

6. (a) Define subring. Prove that intersection of two subrings is a subring. 1+4

(b) Prove that a ring R is commutative if and only if $(a + b)^2 = a^2 + 2ab + b^2$. 4

(c) Define Boolean ring. 1

7. (d) Let the integer $n \geq 2$ and

$Z_n = \{0, 1, 2, \ldots, n - 1\}$. Show that Z_n is a commutative ring with unity under addition and multiplication mod n. 5

(e) A nonempty subset S of ring R is subring of R if and only if $x - y, xy \in S \ \forall \ x, y \in S$. 5

UNIT–IV

8. (a) Prove that the characteristic of an integral domain is either 0 or a prime number. 4

(b) Show that the commutative ring D is an integral domain iff for a, b, c $\in$ D with $a \neq 0$, the relation $ab = ac$ implies that $b = c$. 4

(vii) An integral domain is :

(a) always a field

(b) never a field

(c) a field when it is finite

(d) none of these.

(viii) $p(x) = 2 - 3x + 2x^2 + x^3 \in R[x]$ is :

(a) Associate polynomial

(b) Relatively prime polynomial

(c) Not monic polynomial

(d) Monic polynomial.

(ix) In a polynomial if all its coefficients are integers and its leading coefficient is 1 is called :

(a) Primitive polynomial

(b) Integer monic polynomial

(c) Reducible polynomial

(d) Irreducible polynomial.

(x) In ring R, $x^2 = x$, $\forall\, x \in R$ then R is :

(a) Division ring

(b) Boolean ring

(c) Ring with unity

(d) Commutative ring. 10

UNIT–I

2. (a) Let H be a subgroup of G.. Let $N(H) = \{g \in G \mid gHg^{-1} = H\}$. Show that H is normal in G if and only if $N(H) = G$. 4

(b) Let G be a group in which, for some integer $n > 1$, $(ab)^n = a^n b^n$ for all $a, b \in G$. Show that $G^{(n)} = \{x^n \mid x \in G\}$ is normal subgroup of G. 5

(c) Define Normal Subgroup. 1

3. (d) If N and M are normal subgroup of group G, then prove that NM is also a normal subgroup of G. 4

(e) If a cyclic subgroup N of G is normal in G then prove that every subgroup of N is normal in G. 4

(f) Define :

(i) Quotient Group

(ii) Simple Group. 1+1

UNIT–II

4. (a) If f is homomorphism of a group G into a group G^1, then prove that ker f is a normal subgroup of G. 5

(b) If H, K are normal subgroup of G :

Prove that $\frac{KH}{H} \approx \frac{K}{K \cap H}$. 5