B.Sc. (Part—III) Semester—VI Examination

MATHEMATICS—XII

Graph Theory (Optional)

Time—Three Hours] [Maximum Marks—60

Note :— (1) Question ***one*** is compulsory and attempt it at once only.

(2) Solve ***one*** question from each unit.

1. Choose the correct alternative in the following :

(i) A graph in which all vertices are of equal degree is called as _____.

(a) simple graph

(b) connected graph

(c) regular graph

(d) complete graph

(ii) An open walk in which no vertex appears more than once is _____.

(a) a path

(b) a circuit

(c) disconnected graph

(d) none of these

(iii) A tree with n vertices has _____ edges.

(a) 1 – n

(b) n

(c) n – 1

(d) n + 1

(iv) There are _____ labelled trees with n vertices ($n \geq 2$).

(a) 2n

(b) 2n + 1

(c) n^{n-2}

(d) $(n-2)^n$

(v) Number of edges in the smallest cut set of a connected graph is called as _____.

(a) separability

(b) vertex connectivity

(c) edge connectivity

(d) none

(vi) Sum of degree of regions of plane graph is equal to _____.

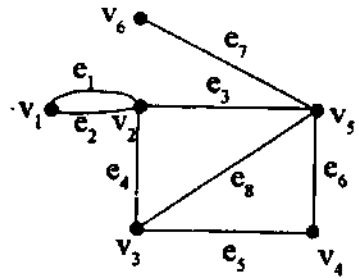
(a) 2n

(b) no. of vertices

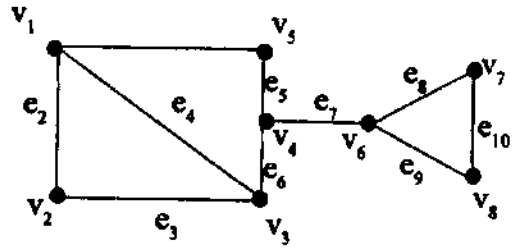
(c) twice the number of edges

(d) none of these

11. (p) Find the cutset matrix of graph G. 5



(q) Find circuit matrix of the graph. 5



(q) Prove that a planar graph with n vertices and e edges has e – n + 2 number of regions. 5

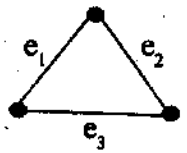
UNIT—IV

8. (a) Prove that the set W_Γ of all circuit vectors including zero vector in W_G forms a subspace of W_G. 5

(b) Prove that the circuit subspace W_Γ and the cutset subspace W_S are orthogonal to each other in a vector space of a graph. 5

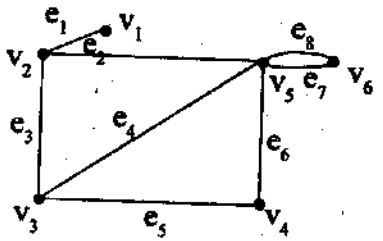
9. (p) Prove that the set W_S of all cutset vectors including zero vector in W_G forms a subspace of W_G. 5

(q) For a graph G with spanning tree $T = \{e_1, e_2\}$ find W_G, W_S, W_Γ, $W_\Gamma \cap W_S$, $W_\Gamma \cup W_S$. 5



UNIT—V

10. (a) Define Incidence matrix and find it for the graph G. 5



(b) Prove that if A(G) is an incidence matrix of a graph G with n vertices then rank of A(G) is (n – 1). 5

(vii) Minimum number of linearly independent vectors that spans the vectors in a vector space W_G is called _____.

(a) dimension of vector space
(b) basis of vector space
(c) subspace
(d) subgraphs

(viii) The dimension of the cutset subspace W_S is equal to the _____.

(a) degree of vertices
(b) no. of edges
(c) rank of the graph
(d) nullity of the graph

(ix) A row with all zeros in incidence matrix represents _____.

(a) pendent vertex
(b) isolated vertex
(c) even vertex
(d) odd vertex

(x) In a path matrix there is no row with all _____.

(a) ones
(b) zeros
(c) vertices
(d) edges

10×1=10

UNIT—I

2. (a) Define degree of a vertex in a graph and show that sum of degrees of all vertices in a graph is twice the number of edges. 5

   (b) Prove that a graph G is disconnected if and only if its vertex set V can be partitioned into two non empty disjoint subsets V_1 and V_2 such that there exist no edge in G whose one end vertex is in subset V_1 and another in V_2. 5

3. (p) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. 5

   (q) Prove that given connected graph G is an Euler graph if and only if all vertices of G are of even degree. 5

UNIT—II

4. (a) Prove that every tree has either one or two centers. 5

   (b) Prove that a graph is a tree if and only if it is minimally connected. 5

5. (p) Prove that following statements are equivalent :

   (i) There is exactly one path between every pairs of vertices in G.

   (ii) G is minimally connected graph. 5

   (q) Prove that a tree with n vertices has n – 1 number of edges. 5

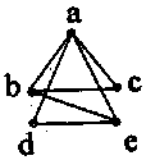
UNIT—III

6. (a) Prove that every circuit has even number of edges in common with the cutset. 5

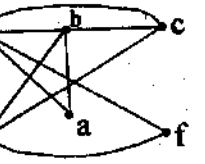
   (b) Define edge connectivity and vertex connectivity and show that the vertex connectivity of any graph can never exceed the edge connectivity. 5

7. (p) Show that following graphs are planar by redrawing it so that there is no edge cross :

   (i)



   (ii)



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