B.Sc. (Part—III) Semester—VI Examination

MATHEMATICS

Paper—XII

(Graph Theory)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory and attempt at once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) A graph $G = < V, E >$ is said to be a null graph if _____.

(a) $V \cap E = \phi$ (b) $V \cup E = \phi$

(c) $V = \phi$ (d) $E = \phi$ 1

(ii) A graph in which all vertices are of equal degree is called _____.

(a) planar graph (b) complete graph

(c) simple graph (d) regular graph 1

(iii) The length of the longest path in a tree is called its _____.

(a) centre (b) radius

(c) diameter (d) walk 1

(iv) If a tree has n vertices then number of labelled trees are _____.

(a) n^{n-2} (b) n^2

(c) n^{n-1} (d) n^{n+1} 1

(v) A complete graph of five vertices is called as :

(a) planar graph (b) non-planar graph

(c) vertex graph (d) biparticle graph 1

(vi) A connected graph G is a tree iff adding an edge between any two vertices in G creates exactly _____.

(a) one circuit (b) two circuits

(c) three circuits (d) many circuits 1

(vii) Subspaces W_Γ and W_S are said to be orthogonal complements iff _____.

(a) dim $(W_\Gamma \cup W_S) = 0$ (b) dim $(W_\Gamma \cap W_S) = 0$

(c) dim $(W_\Gamma \cup W_S) = 1$ (d) dim $(W_\Gamma \cap W_S) = 1$ 1

(viii) In a cutset matrix C(G) a column with all zeros corresponds to an edge forming _____.

(a) a tree (b) a binary tree

(c) a loop (d) the parallel edges 1

(ix) If B(G) is a circuit matrix in a connected graph G with n vertices and e edges, then rank of B(G) = _____.

(a) e – n + 1 (b) e – n + 1

(c) n – 1 (d) n + 1 1

(x) If no vertex appears more than once in an edge sequence then it is called as _____.

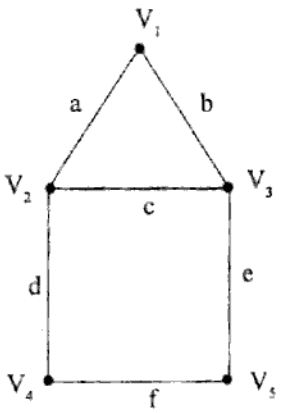
(a) a circuit (b) a cutset

(c) a walk (d) a path 1

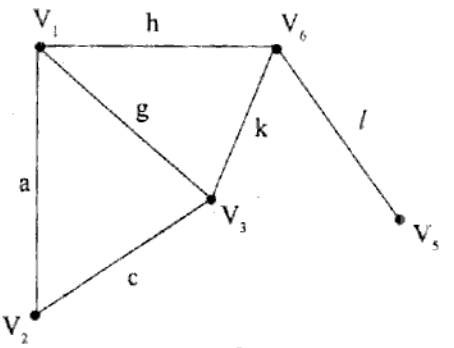
UNIT—I

2. (a) A connected graph G is an Euler graph iff every vertex of G is an even degree, prove this. 5

(b) Define union and intersection of two graphs G_1 and G_2. From the following figures find : (i) $G_1 \cup G_2$ (ii) $G_1 \cap G_2$ (iii) $G_1 \oplus G_2$.



(G_1)



(G_2)

5

3. (p) Define complete graph. Draw complete graph of four and five vertices. Also prove that the number of vertices of odd degree in a graph is always even. 5

(q) Define graph and draw the graphs of the following chemical compounds :

(i) CH_4 (ii) C_2H_6

(iii) C_6H_6 (iv) N_2O_3 5

UNIT—II

4. (a) Define tree. If G is a graph with n vertices then prove that following statements are equivalent :

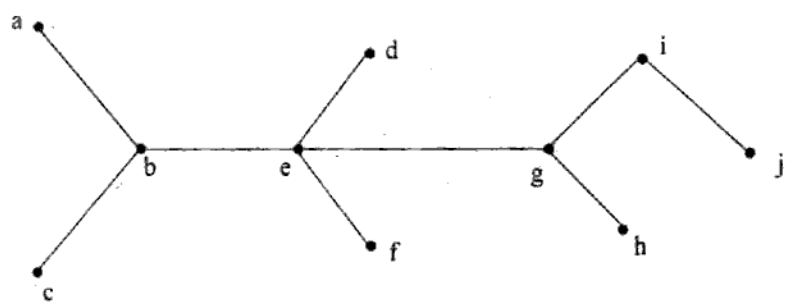
(i) G is a tree.

(ii) G is connected and has n – 1 edges. 5

(b) Prove that in any tree with two or more vertices there are at least two pendant vertices. 5

5. (p) Prove that a graph T is a tree if and only if there is only one path between every pair of vertices in T. 5

(q) Redraw the tree given below as Rooted tree with b as a root. What is the height of resulting tree ? Find Centre(s), radius and diameter of tree.



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UNIT—III

6. (a) Define the region and prove that if G is a planar graph with n vertices, e edges and f regions then n – e + f = 2. 5

(b) Prove that ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets. 5

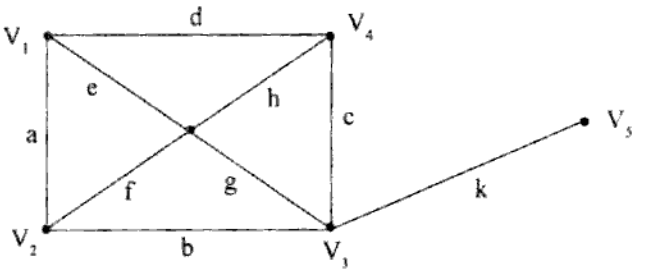
7. (p) Define :—

(i) Fundamental circuit

(ii) Fundamental cut set.

Prove that with respect to a given spanning tree T, a branch b_r that determines a fundamental cut set S is contained in every fundamental circuit associated with the chords in S and in no others. 5

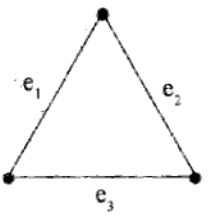
(q) Define a cut set. List all the cut sets in the following graph : 5



UNIT—IV

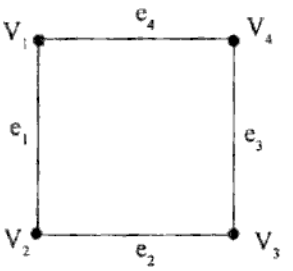
8. (a) In the vector space of a graph prove that the circuit subspace and the cut-set subspace are orthogonal to each other. 5

(b) For the given graph G, find IN_G, W_S, W_Γ, $W_S \cap W_\Gamma$ are $W_S \cup W_\Gamma$. 5



9. (p) Prove that the set W_S of all cut-set vectors including zero vector in W_G form a subspace of W_G. 5

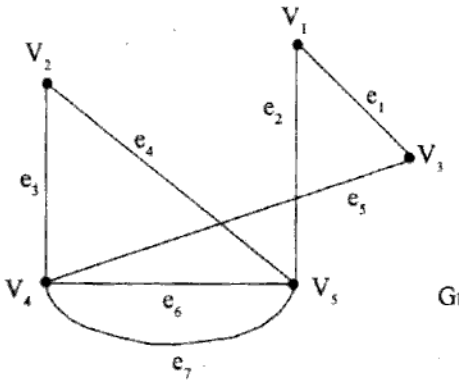
(q) Let G be a graph given as in figure. Find W_Γ, W_S, $W_\Gamma \cap W_S$ and $W_\Gamma \cup W_S$, where W_Γ is a circuit subspace and W_S is a cut-set subspace. 5



Graph G

UNIT—V

10. (a) Find Incidence matrix A(G) and the Adjacency matrix of the following graph G.



Graph G

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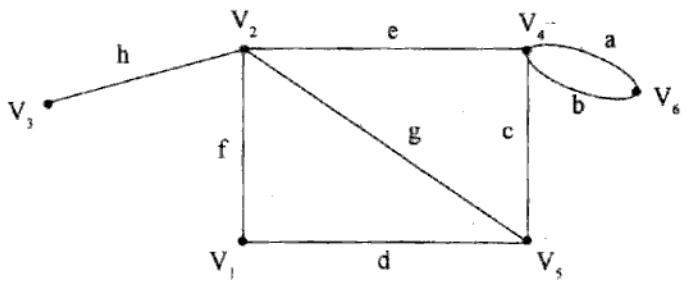
(b) If A(G) is an incidence matrix of connected graph G with n vertices then prove that rank of A(G) is (n – 1). 5

11. (p) Let A and B be respectively, the incidence matrix and the circuit matrix of a loop free graph whose columns are arranged using the same order of edges. Then show that every row of A is orthogonal to every row of B, i.e. $AB^T = 0$, $B \cdot A^T = 0 \pmod 2$. 5

(q) Define circuit matrix. Find the circuit matrix of the following graph G.



Graph G

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