AT–399

**B.Sc. (Part-III) Semester-VI Examination**

**MATHEMATICS**

**Graph Theory (Optional)**

**Paper—XII**

Time : Three Hours] [Maximum Marks : 60

**Note** :— (1) Question No. **1** is compulsory and attempt it at once only.

(2) Solve **ONE** question from each unit.

1. Choose the correct alternative in the following :

(1) A simple graph in which if every two distinct vertices are adjacent then it is said to be : 1

(a) Connected graph (b) Regular graph

(c) Null graph (d) Complete graph

(2) The vertex with zero degree is called as : 1

(a) Even vertex (b) Odd vertex

(c) Pendant vertex (d) Isolated vertex

(3) There are $\eta^{n-2}$ labelled tree's with n vertices ($n \geq 2$). This is defined by : 1

(a) Euler formula (b) Cayley formula

(c) Hamiltonian formula (d) Kuratowski formula

(4) In any tree with one or more vertices, there are at least : 1

(a) One pendant vertex (b) One isolated vertex

(c) Two pendant vertices (d) Two isolated vertices

(5) A connected graph is said to be separable if its vertex connectivity is : 1

(a) 1 (b) 2

(c) 3 (d) 4

(6) A graph can be embedded in the surface of a sphere iff it can be embedded in : 1

(a) a circle (b) a straight line

(c) a sphere (d) a plane

(7) Minimum number of linearly independent vectors that spans the vectors in a vector space $W_G$ is called : 1

(a) Subspace (b) Dimension of vector space

(c) Subgraphs (d) Basis of vector space

(8) A row with a single unit entry in incidence matrix corresponds to : 1

(a) a isolated vertex (b) an internal vertex

(c) a pendant vertex (d) none of these

(9) If B is a circuit matrix of a connected graph G with n vertices and e edges then rank of B is : 1

(a) e + n − 1 (b) e − n − 1

(c) e + n + 1 (d) e − n + 1

(10) The number of pendant vertices in binary tree with 15 vertices are : 1

(a) 7 (b) 8

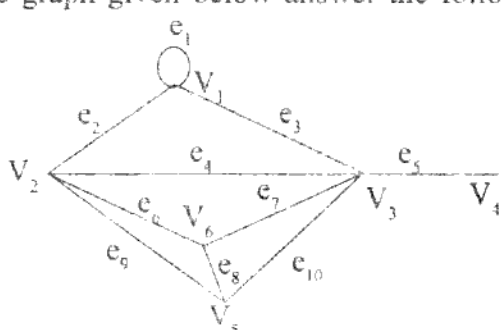
(c) 9 (d) 10

## UNIT—I

2. (a) Define degree of a vertex. Prove that in a simple graph there are even number of odd degree vertices. 1+4

(b) Define a graph. Prove that a simple graph with n vertices and k-component can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. 1+4

3. (p) From the graph given below answer the following :




(i) Write the adjacent vertices of $V_6$.

(ii) Which edges are incident with the vertex $V_2$ ?

(iii) Write the degree of each vertex.

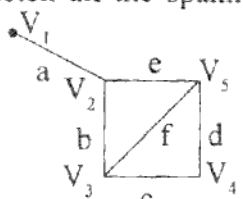
(iv) Is the graph simple ? Why ? 1+1+1+2

(q) Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits. 5

## UNIT—II

4. (a) Prove that in any tree with two or more vertices there are atleast two pendant vertices. 5

(b) Define spanning tree. Sketch all the spanning tree of a graph given below. 2+3



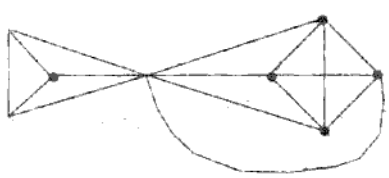

5. (p) Prove that a graph is a tree if and only if it is minimally connected. 5

(q) Define centre of a tree and show that every tree has either one or two centres. 5

## UNIT—III

6. (a) Prove that a connected planar graph with n vertices and e edges has e – n + 2 region. 5

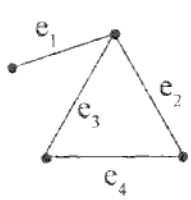
(b) Define vertex connectivity and edge connectivity of connected graph. What is the vertex connectivity and edge connectivity of the following graph ?



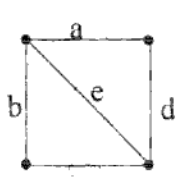
Draw a graph with same number of vertices and edges of above graph so that vertex connectivity and edge connectivity should be maximum. 1+1+1+1+1

7. (p) Show that the complete graph with five vertices is non-planar. 5

(q) Define cut-set of a graph. Find all cut-set of the following graphs. 1+2+2




Graph $G_1$

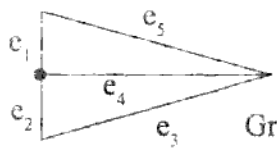



Graph $G_2$

## UNIT—IV

8. (a) Prove that the circuit subspace $W_F$ and the cut-set subspace $W_S$ are orthogonal to each other in vector space of a graph. 5

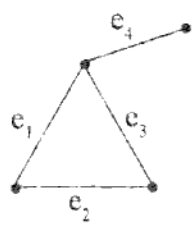
(b) Find all circuits and cutsets of the graph G given below and calculate $W_\Gamma$ and $W_S$ and their dimensions. 5



Graph G

9. (p) Prove that $W_S$ of all cut-set vectors including zero vector, in $W_G$ forms a subspace of $W_G$. 5

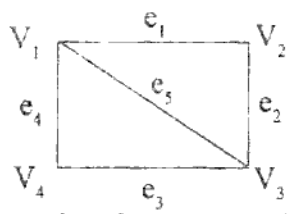
(q) For the following graph, find $W_\Gamma \cup W_S$ and $W_\Gamma \cap W_S$ of spanning tree $T = \{e_1, e_2, e_4\}$. 5




## UNIT—V

10. (a) The reduced incedence matrix of a graph is non-singular iff the graph is tree. 5

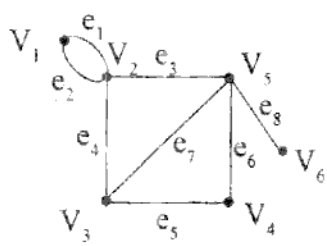
(b) For the following G find A(G), B(G) and prove that $AB^T$ = zero matrix. 2+2+1



Graph G

11. (p) If A(G) is an incidence matrix of a connected graph G with n vertices then prove that the rank of A(A) is n – 1. 5

(q) Define cut-set matrix. Find the cut-set matrix of the graph G. 1+4



Graph G