B.Sc. Part–III (Semester–VI) Examination

MATHEMATICS (New)

Paper—XII

Graph Theory (Optional)

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it at once only.

(2) Solve **ONE** question from each unit.

1. Choose the correct alternatives in the following :— 1 mark each

(1) The number of vertices of odd degree in a graph is always _____.

(a) odd (b) even

(c) even and odd (d) none of these

(2) For any graph G with e edges and n vertices sum of degree of all vertices is equal to _____.

(a) 2e (b) e – 2

(c) e + 1 (d) e – 1

(3) An edge which is in a spanning tree T is called as _____.

(a) branch (b) chord

(c) cutset (d) circuit

(4) The total number of pendant vertices in a binary tree with n vertices are _____.

(a) n – 1 (b) n + 1

(c) $\frac{n-1}{2}$ (d) $\frac{n+1}{2}$

(5) Number of edges in the smallest cutset of a connected graph is called as _____.

(a) vertex connectivity (b) edge connectivity

(c) separability (d) none of these

(6) Every cut-set in a non separable graph with more than two vertices contains at least _____.

(a) one edge (b) two edges

(c) three edges (d) none of these

(7) The circuit subsapce W_Γ and the cutset subspace W_s in the vector space of a graph are _____.

(a) orthogonal to each other

(b) parallel to each other

(c) not orthogonal to each other

(d) not parallel to each other

(8) The dimension of the cutset subsapce W_s is equal to the _____.

(a) degree of vertices (b) no. of edges

(c) rank of the graph (d) nullity of the graph

(9) If A(G) is an incidence matrix of a connected graph G with n vertices then rank of A(G) is _____.

(a) $\frac{n+1}{2}$ (b) $\frac{n-1}{2}$

(c) $n + 1$ (d) $n - 1$

(10) In a path matrix there is no row with all _____.

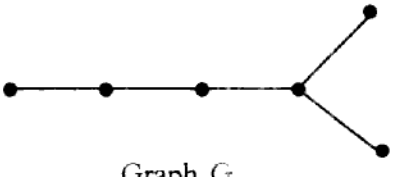
(a) zeros (b) ones

(c) vertices (d) edges

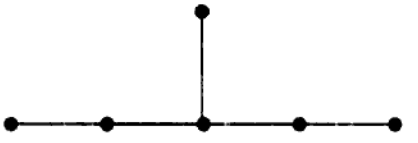
UNIT—I

2. (a) Define a simple graphs and show that maximum number of edges in a simple graph with n vertices are $\frac{n(n-1)}{2}$. 1+4

(b) When two graph are said to be isomorphic ? Whether the following graphs are isomorphic or not ? Explain.



Graph G_1



Graph G_2

1+4

3. (p) Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits. 5

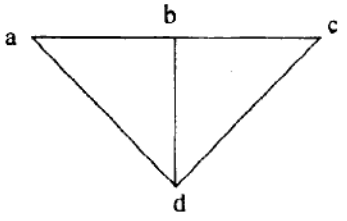
(q) Define degree of a vertex. Show that in a connected graph there are even number of odd degree vertices. 1+4

UNIT—II

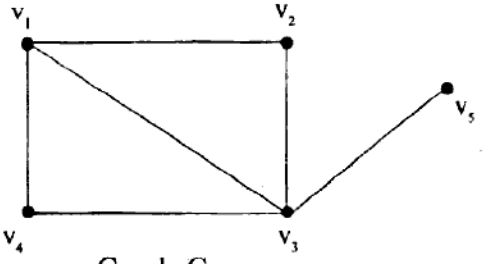
4. (a) Define centre of a tree and show that every tree has either one or two centres. 1+4

(b) Define binary tree and rooted tree. Show that there are $\frac{n+1}{2}$ pendant vertices in a binary tree with n vertices. 1+1+3

5. (p) Sketch all spanning trees of the following graphs G_1 and G_2.



Graph G_1



Graph G_2

5

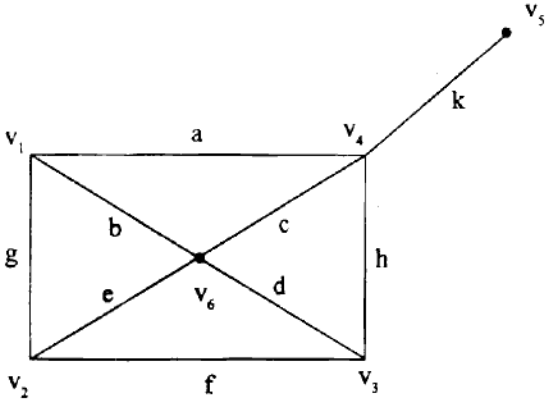
(q) Define distance between two vertices in connected graph. Prove that the distance between two vertices in connected graph is a metric. 1+4

UNIT—III

6. (a) Prove that the complete graph of five vertices is nonplanar. 5

(b) Prove that if G is a planar connected graph with n vertices, e edges and f faces (region) then $n - e + f = 2$. 5

7. (p) Define cutset. List all the cutsets in the following graph.



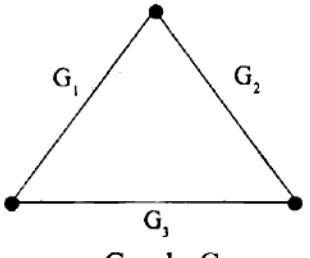
2+3

(q) Prove that every cutset in a connected graph G must contain at least one branch of every spanning tree of a graph G. 5

UNIT—IV

8. (a) Prove that the set W_Γ of all circuit vectors including zero vector in W_G forms a subspace of W_G. 5

(b) For the graph G find W_G, W_S, W_Γ, $W_S \cap W_\Gamma$ and $W_S \cup W_\Gamma$.



Graph G

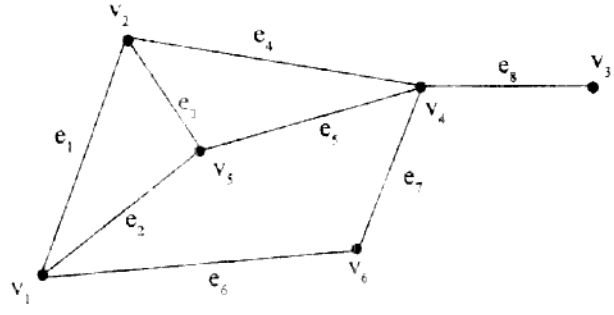
5

9. (p) Prove that the set of circuit vector corresponding to the set of fundamental circuits, with respect to any spanning tree. forms a basis for the circuit subsapce W_f. 5

(q) Prove that the dimension of the cutset subspace W_S is equal to the rank r of the graph and the number of cutset vectors (including o) in W_S is 2^r. 5

UNIT—V

10. (a) Define adjacency matrix. Find the adjacency matrix of the graph G.



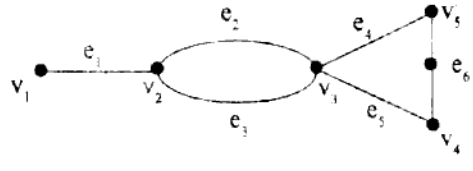
Graph G

5

(b) If B is a circuit matrix of a connected graph G with e edges and n vertices then prove that rank of B = e – n + 1. 5

11. (p) Prove that the reduced incidence matrix of a graph is nonsingular iff the graph is a tree. 5

(q) Find incidence matrix A(G) and cut set matrix C(G) for the graph G.



Graph G

5