B.Sc. (Part—III) Semester—VI Examination
MATHEMATICS (Old) (Upto Winter—2018)
(Special Theory of Relativity)
Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Solve **ONE** question from each unit.

1. Choose the correct alternatives :

(i) If $ds^2 > 0$, then the interval 'ds' is said to be :

(a) Light like (b) Space like

(c) Time like (d) None of these 1

(ii) The electromagnetic field tensor F_{ij} is antisymmetric if :

(a) $F_{ij} = F_{ji}$ (b) $F_{ij} = -F_{ji}$

(c) $F_{ij} = -F_{ij}$ (d) None of these 1

(iii) If p is the magnitude of momentum of a moving particle and E is the energy, then the numerical value of $p^2 - \frac{E^2}{c^2}$ is :

(a) $m_0^2 c^2$ (b) $-m_0^2 c^2$

(c) $m_0^2 c$ (d) $m_0 c^2$ 1

(iv) If $\overline{A}$ is a vector potential, then the magnetic field is given by :

(a) $\overline{H} = \nabla\phi \times \overline{A}$ (b) $\overline{H} = \text{div}.\overline{A}$

(c) $\overline{H} = \text{curl}\,\overline{A}$ (d) None of these 1

(v) Number of components of a tensor T_{ij} in four dimensions is :

(a) 10 (b) 6

(c) 16 (d) None of these 1

(vi) The mass of particle $m = \frac{m_0}{\sqrt{1 - \frac{u^2}{c^2}}}$, where the particle is moving with velocity u relative to inertial frame is called as :

(a) Equivalent mass of a particle (b) Relativistic mass of a particle

(c) Inertial mass of a particle (d) None of these 1

(vii) Principle of relativity means :

(a) Some inertial frames are equivalent (b) All inertial frames are equivalent

(c) Inertial frames are not equivalent (d) None of these 1

(viii) The time recorded by a clock moving with a body is called as :

(a) Absolute time (b) Proper time

(c) Improper time (d) None of these 1

(ix) If an electromagnetic field is purely magnetic in an inertial frame, then the field in s′ is :

(a) Only electric (b) Only magnetic

(c) Electric as well as magnetic (d) None of these 1

(x) Maxwell's equations of electromagnetic theory are invariant under :

(a) Galilean transformations (b) Lorentz transformations

(c) Guage transformations (d) None of these 1

UNIT—I

2. (a) Show that Lorentz transformation forms a group with respect to multiplication. 4

(b) Show that the three dimensional volume element dxdydz is not Lorentz invariant but the four dimensional volume element dxdydzdt is Lorentz invariant. 3

(c) Show that simultaneity is relative in special relativity. 3

3. (p) Prove that in an inertial frame a body without influence of any forces moves in a straight line with constant velocity. 3

(q) Show that $x^2 + y^2 + z^2 - c^2t^2$ is Lorentz invariant. 3

(r) Discuss the geometrical interpretation of Lorentz transformations. 4

UNIT—II

4. (a) Obtain the transformations for the velocities of particle under special Lorentz transformations. 5

(b) Prove that if the body moves with uniform velocity v relative to the observer, then its apparent length is contracted by the factor $\left(1 - \frac{u^2}{c^2}\right)^{1/2}$ in the direction of relative motion. 5

5. (p) Obtain the transformation of the Lorentz contraction factor $\left(1-\frac{u^2}{c^2}\right)^{1/2}$. 5

(q) Let u and u′ be the velocities of a particle in two inertial system s and s′ respectively, where s′ is moving with velocity v relative to s along xx′ axis. Show that :

$$\tan\theta' = \frac{\sin\theta\left(1-\frac{v^2}{c^2}\right)^{1/2}}{\cos\theta - \frac{v}{u}}$$

$$\text{and } u'^2 = \frac{u^2\left[1-2\frac{v}{u}\cos\theta+\left(\frac{v}{u}\right)^2-\left(\frac{v}{u}\right)^2\sin^2\theta\right]}{\left(1-\frac{uv}{c^2}\cos\theta\right)^2}. \quad 5$$

UNIT—III

6. (a) Define :
   (i) Time-like interval
   (ii) Space-like interval
   (iii) Four vector A^r
   (iv) Proper time. 4

   (b) Prove that there exists an inertial system s' in which the two events occur at one and the same time if the interval between two events is spacelike. 4

   (c) What do you mean by covariant and contravariant vector ? 2

7. (p) Show that :

   $x^1 = -x_1,\ x^2 = -x_2,\ x^3 = -x_3,\ x^4 = x_4$

   and hence $x_i = (-\bar{r}, ct)$. 4

   (q) Show that the square of the length of a four vector is invariant under Lorentz transformations. 3

   (r) Obtain the metric of the space-time geometry of special relativity. 3

UNIT—IV

8. (a) Define four force. Prove that the four force in component form is expressed as :

$$f^i = \left(\frac{\bar{F}}{c\sqrt{1-\frac{u^2}{c^2}}}, \frac{\bar{F}\cdot\bar{u}}{c^2\sqrt{1-\frac{u^2}{c^2}}}\right), \text{ where } \bar{F} = \frac{d\bar{p}}{dt}. \quad 1+3$$

   (b) Prove that $E = c\sqrt{p^2 + m_0^2c^2}$ and $\frac{dE}{dp} = u$. 3

   (c) Define four velocity and four acceleration. Show that the four velocity of a particle is a unit timelike vector. 1+1+1

9. (p) Obtain the transformation equations for momentum and energy of a particle. 5

(q) Obtain the mass energy equivalence relation $E = mc^2$, where m is the relativistic mass of the particle. 5

UNIT—V

10. (a) Define Four potential. Write the transformations of the electromagnetic four potential vector using Lorentz transformations. 1+4

(b) Prove that the set of Maxwell's equations $\text{div}.\overline{E} = 0$ and $\text{curl}\ \overline{H} = \frac{1}{c}\frac{\partial \overline{E}}{\partial t}$ can be written as

$\frac{\partial F_{ik}}{\partial x^k} = 0$, where F_{ik} is the electromagnetic field tensor. 5

11. (p) Define electromagnetic field tensor F_{ij}. Express the components of F_{ij} in terms of the electric and magnetic field strengths. 1+4

(q) Define Electric field strength $\overline{E}$ and Magnetic field strength $\overline{H}$ in terms of scalar and vector potential. Show that $\overline{E}$ and $\overline{H}$ remain invariant under Guage transformations. 1+1+3