B.Sc. (Part—III) Semester—VI Examination
MATHEMATICS (New)
(Special Theory of Relativity)
Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory.
(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) The order of outer product is the _____ of the order of the tensors.
(a) Product (b) Difference
(c) Sum (d) None of these 1

(ii) The interval ds is said to be time like if :
(a) $ds^2 = 0$ (b) $ds^2 < 0$
(c) $ds^2 > 0$ (d) None of these 1

(iii) In Newtonian Mechanics, an event is identified by _____ real numbers.
(a) 1 (b) 2
(c) 3 (d) 4 1

(iv) 'Principle of Relativity' means :
(a) Some inertial frame are equivalent (b) All inertial frame are not equivalent
(c) All inertial frame are equivalent (d) None of these 1

(v) Length contraction means :
(a) Moving rod measures shorter (b) Moving rod measures larger
(c) Rest rod measures shorter (d) Rest rod measures longer 1

(vi) In relativistic addition law for velocities, when $c \to \infty$ then :
(a) $u' = u + v$ (b) $u' = u - v$
(c) $u' = v - u$ (d) None of these 1

(vii) Four velocity of a particle is defined as :
(a) $u^i = \frac{ds}{dx^i}$ (b) $u^i = \frac{dx^i}{ds}$
(c) $u = \frac{dx}{ds^i}$ (d) $u = \frac{dx^i}{ds}$ 1

(viii) $\bar{F}$ = mass X acceleration where mass = _____ is the longitudinal mass of the particle.
(a) $\frac{m_0}{(1 - u^2/c^2)^{1/2}}$ (b) $\frac{m_0}{(1 - u^2/c^2)^{3/2}}$
(c) $\frac{m_0}{(1 - u^2/c^2)^{-3/2}}$ (d) None of these 1

(ix) Mass energy equivalence relation is given by :

(a) $E = mc^2$ (b) $E = m/c^2$

(c) $E = c^2/m$ (d) None of these 1

(x) If $\overline{A}$ is a vector potential then magnetic field is given by :

(a) $\overline{H} = \text{div } \overline{A}$ (b) $\overline{H} = \text{curl } \overline{A}$

(c) $\overline{H} = \Delta\phi \times A$ (d) None of these 1

UNIT—I

2. (a) Obtain Galilean transformation equation for two inertial frames in relative motion. 4

(b) Show that the circle $x'^2 + y'^2 = a^2$ in s' is measured to be an ellipse in s if s' moves with uniform velocity relative to s. 2

(c) Show that the Newton Kinematical equations of motion are invariant under Galilean transformation. 4

3. (a) What are Lorentz transformations ? Obtain an expression for them. 6

(b) Prove that in an inertial frame a body without influence of any forces, moves in a straight line with constant velocity. 4

UNIT—II

4. (a) Show that the velocities u and u' measured in two inertial systems s and s' are related by

$$\sqrt{1-\frac{u^2}{c^2}} = \frac{\sqrt{1-\frac{u'^2}{c^2}}\cdot\sqrt{1-\frac{v^2}{c^2}}}{\left(1+\frac{u'_x v}{c^2}\right)}$$

where s' is moving with velocity 'v' relative to s along xx' axis. 5

(b) Show that in nature no signal can move with a velocity greater than the velocity of light relative to any inertial system. 5

5. (a) Deduce the transformation of particle velocities and hence obtain relativistic addition law for velocities. 6

(b) Write a short note on 'Time dilation'. 4

UNIT—III

6. (a) Show that the interval or metric ds^2 between two events is given by :

$$ds^2 = -dx^2 - dy^2 - dz^2 + c^2dt^2$$

Prove that ds^2 is invariant under Lorentz transformation. 6

(b) Prove that there exists an inertial system s' in which the two events occur at one and the same point if the interval between two events is time like. 4

7. (a) Prove that :

(i) $T'^{14} = \alpha^2 \left\{ -\frac{v}{c} T^{11} + T^{14} + \frac{v^2}{c^2} T^{41} - \frac{v}{c} T^{44} \right\}$

(ii) $T'^{23} = T^{23}$. 6

(b) Define :

(i) Contravariant tensor of order one

(ii) Covariant tensor of order one

(iii) Kronecker delta

(iv) Space like interval. 4

UNIT—IV

8. (a) Prove that $E = mc^2$, where E is the energy of the particle. 6

(b) Show that the four velocity, in component form can be expressed as :

$$u' = \left(\frac{\bar{u}}{c\sqrt{1 - u^2/c^2}}, \frac{1}{\sqrt{1 - u^2/c^2}} \right), \text{ where } \bar{u} = (u_x, u_y, u_z). \quad 4$$

9. (a) Show that the quantity $p^2 - E^2/c^2$ is an invariant whose numerical value is $-m_0^2 c^2$. 4

(b) Define four momentum vector. Obtain the transformation equations for four momentum and energy. 6

UNIT—V

10. (a) Define current four vector. Transform its components under Lorentz transformation. Deduce an expression $c^2\rho^2 - J^2 = \rho_0^2 c^2 = \text{invariant}$. 6

(b) Obtain the wave equation for the propagation of electric $\bar{E}$ and magnetic $\bar{H}$ field strengths in vacuum with velocity of light. 4

11. (a) Show that the Hamiltonian for a charged particle moving in an electromagnetic field is :

$$H = \left\{ m_0^2 c^4 + c^2 \left(p + \frac{e}{e} A \right)^2 \right\}^{1/2} + e\phi. \quad 5$$

(b) Define electromagnetic field tensor F_{ij} and obtain the components F_{23}, F_{31}, F_{12}, also show that F_{ij} is antisymmetric. 5