(b) If an electromagnetic field is purely magnetic in an inertial frame S, then describe the field in inertial frames S'. 4

11. (p) Obtain the transformation for electric and magnetic field strengths. 5

(q) Define electromagnetic field tensor F_{ij}. Express the components of F_{ij} in terms of the electric and magnetic field strengths. 5

B.Sc. (Part—III) Semester—VI Examination

MATHEMATICS

Paper—XII

(Special Theory of Relativity)

Time—Three Hours] [Maximum Marks—60

Note :— (1) Question ***one*** is compulsory and attempt it at once only.

(2) Solve ***one*** question from each unit.

1. Choose correct alternatives :

(i) Principle of relativity means _____. 1

(a) Some inertial frames are equivalent

(b) All inertial frames are equivalent

(c) inertial frames are not equivalent

(d) None of these

(ii) If $ds^2 = 0$ then the interval 'ds' is said to be : 1

(a) Light like

(b) Space like

(c) Time like

(d) None of these

(iii) Length contraction means _____. 1

(a) Moving rod measures longer

(b) Moving rod measures shorter

(c) Rest rod measures shorter

(d) Rest rod measures longer

(iv) Force $\bar{F}$ = mass × acceleration; where mass = _____ is longitudinal mass of the particle. 1

(a) $\frac{m_0}{(1-u^2/c^2)^{1/2}}$

(b) $\frac{m_0}{(1-u^2/c^2)^{3/2}}$

(c) $\frac{m_0}{(1-u^2/c^2)^{-3/2}}$

(d) None of these

(v) If ϕ is a scalar potential and $\bar{A}$ is the vector potential then the electric field is given by : 1

(a) $\bar{E} = \text{grad}\ \phi - \frac{1}{c}\frac{\partial \bar{A}}{\partial t}$

(b) $\bar{E} = \text{grad}\ \phi + \frac{1}{c}\frac{\partial \bar{A}}{\partial t}$

(c) $\bar{E} = -\text{grad}\ \phi - \frac{1}{c}\frac{\partial \bar{A}}{\partial t}$

(d) $\bar{E} = -\text{grad}\ \phi + \frac{1}{c}\frac{\partial \bar{A}}{\partial t}$

UNIT—IV

8. (a) Prove that $L = -m_0c^2\sqrt{1-u^2/c^2}$ for relativistic Lagrangian 5

(b) Show that $E = mc^2$, where E is the energy of the particle. 5

9. (p) Prove that the four velocity in component form can be expressed as

$$u^i = \left(\frac{\bar{u}}{c\sqrt{1-u^2/c^2}}, \frac{1}{\sqrt{1-u^2/c^2}}\right),$$

where $\bar{u} = (u_x, u_y, u_z)$ = ordinary three dimensional velocity of the particle. 3

(q) Show that the quantity $P^2 - \frac{E^2}{c^2}$ is an invariant whose numerical value is $-m_0c^2$. 4

(r) A particle is given a kinetic energy equal to n times its rest energy m_0c^2. What are (i) its speed and (ii) momentum ? 3

UNIT—V

10. (a) Define electric $\vec{E}$ and magnetic $\vec{H}$ field strengths in terms of scalar and vector potential. Show that $\vec{E}$ and $\vec{H}$ remain invariant under Gauge transformation. 2+4

(q) Prove that if the body moves with uniform velocity v relative to the observer then its apparent length is contracted by the factor $\left(1-\frac{v^2}{c^2}\right)^{1/2}$ in the direction of relative motion. 5

UNIT—III

6. (a) Show that the interval or metric ds^2 between two events is given by :

$$ds^2 = -dx^2 - dy^2 - dz^2 + c^2dt^2.$$

Prove that ds^2 is invariant under Lorentz transformations. 2+3

(b) Define four vectors :

Show that $A^1 = -A_1$, $A^2 = -A_2$, $A^3 = -A_3$ and $A^4 = A_4$. 1+4

7. (p) Define :

(i) Contravariant tensor of rank two

(ii) Kronecker delta

(iii) Covariant tensor of rank two

(iv) Conjugate metric tensor. 4

(q) Obtain the transformations of the components T'^{12} and T'^{13}. 6

(vi) Number of components of a tensor T_{ij} in four dimensions is : 1

(a) 10

(b) 6

(c) 16

(d) None of these

(vii) When u, v << c then the transformation of acceleration is : 1

(a) $\bar{a}' = \bar{a}$

(b) $\bar{a}' < \bar{a}$

(c) $\bar{a}' > \bar{a}$

(d) None of these

(viii) Inertial system means the reference system where _____ is valid. 1

(a) Newton's first law of motion

(b) Newton's second law of motion

(c) Newton's third law of motion

(d) None of these

(ix) Maxwell tensor F_{ij} is _____. 1

(a) $\frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}$

(b) $\frac{\partial A_j}{\partial x^i} - \frac{\partial A_i}{\partial x^j}$

(c) $\frac{\partial A_i}{\partial x^j} + \frac{\partial A_j}{\partial x^i}$

(d) None of these

(x) The mass of particle $m = \frac{m_0}{\sqrt{1-u^2/c^2}}$; where the particle is moving with velocity u relative to inertial frame is called as _____. 1

(a) Equivalent mass of a particle

(b) Relativistic mass of a particle

(c) Rest mass of a particle

(d) None of these

UNIT—I

2. (a) Show that the electromagnetic wave equation :

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}=0$$

is not invariant under the Galilean transformation equations. 4

(b) Define inertial system. Prove that in an inertial frame a body, without influence of any forces, moves in a straight line with constant velocity. 4

(c) Show that $x^2 + y^2 + z^2 - c^2t^2$ is Lorentz invariant. 2

3. (p) Discuss the geometrical interpretation of Lorentz transformations. 4

(q) Prove that Newton's kinematical equations of motion are invariant under the Galilean transformations. 3

(r) Show that the three dimensional volume element dxdydz is not Lorentz invariant but the four dimensional volume element dxdydzdt is Lorentz invariant. 3

UNIT—II

4. (a) Deduce the transformations of particle velocities and hence obtain relativistic addition law for velocities. 6

(b) There are three galaxies G_1, G_2 and G_3. Observations in G_1 show that G_2 and G_3 are moving in opposite directions each with a speed of 0.5 c. What is the speed of G_1 observed in G_2 ? What is the speed of G_3 measured in G_2 ? 4

5. (p) Show that in nature no signal can move with a velocity greater than the velocity of light relative to any inertial system. 5