AS–1473

B.Sc. (Part—III) Semester—VI Examination

MATHEMATICS

(Special Theory of Relativity)

Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory.

(2) Attempt **one** questions from each unit.

1. Choose the correct alternative : 1

(i) The reference system is said to be an inertial system if :

(a) Newton's first law of motion valid

(b) Newton's second law of motion valid

(c) Newton's third law of motion valid

(d) None of these.

(ii) The special Lorentz transformations will reduce to simple Galilean transformations when : 1

(a) $V = C$ (b) $V >> C$

(c) $V << C$ (d) None of these

(iii) The simultaneity in special relativity is : 1

(a) relative (b) constant

(c) absolute (d) None of these

(iv) The time recorded by a clock moving with a body is known as : 1

(a) Time dilation (b) Proper time

(c) Fixed time (d) None of these

(v) The interval ds is said to be time-like if : 1

(a) $ds^2 = 0$ (b) $ds^2 < 0$

(c) $ds^2 > 0$ (d) None of these

(vi) Mass energy equivalence relation is given by : 1

(a) $E = mc^2$ (b) $E = m/c^2$

(c) $E = c^2/m$ (d) None of these

(vii) Four velocity of a particle is defined as :

(a) $u^i = \frac{ds}{dx^i}$ (b) $u^i = \frac{dx^i}{ds}$

(c) $u = \frac{dx}{ds^i}$ (d) $u = \frac{dx^i}{ds}$

(viii) If $\overline{A}$ is a vector potential then the magnetic field is given by : 1

(a) $\overline{H} = \text{div } \overline{A}$ (b) $\overline{H} = \text{curl } \overline{A}$

(c) $\overline{H} = \Delta\phi \times A$ (d) None of these

(ix) The electric and magnetic field strengths remain invariant under : 1

(a) Galilean transformations (b) Gauge transformations

(c) Fourier transformations (d) None of these

(x) The transformations $\overline{r}^1 = \overline{r} - \overline{v}t$ and $t^1 = t$ are known as : 1

(a) General Lorentz transformations (b) Special Lorentz transformations

(c) Simple Galilean transformations (d) General Galilean transformations

UNIT—I

2. (a) Discuss the Geometrical interpretations of Lorentz transformations. 4

(b) Prove that $\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is invariant under special Lorentz transformations. 4

(c) What are the postulates of the special theory of relativity ? 2

3. (a) Obtain Galilean transformation equations for two inertial frames in relative motion. 4

(b) Show that the Newton's kinematical equations of motion are invariant under Galilean transformations. 4

(c) Show that the circle $x'^2 + y'^2 = a^2$ in S' is measured to be an ellipse in S if S' moves with uniform velocity relative to S. 2

UNIT—II

4. (a) Deduce the transformations of particle velocities and hence obtain relativistic addition law for velocities. 6

(b) Obtain the transformation of the Lorentz contraction factor $\sqrt{1 - u^2/c^2}$ 4

5. (a) If u and u' are the velocities of a particle measured in the frames S and S' respectively, then obtain the expressions a'_x, a'_y and a'_z for acceleration of a particle. 6

(b) An observer moving along the x-axis of S with velocity V observes a body of proper volume V_0 moving with velocity u along the x axis of S. Show that the observer measures the volume to be equal to $V_0\sqrt{\frac{(c^2 - v^2)(c^2 - u^2)}{(c^2 - uv)^2}}$. 4

UNIT—III

6. (a) Obtain the metric $ds^2 = -dx^2 - dy^2 - dz^2 + c^2 dt^2$ of the space time geometry of special relativity. Prove that ds^2 is invariant of special relativity. Prove that ds^2 is invariant under the Lorentz transformations. 5

   (b) Define time-like and space-like intervals. Prove that there exists an inertial system S' in which two events occur at one and the same point if the interval between two events is time-like. 5

7. (a) Obtain the transformations of the components T'^{11} and T'^{14}. 6

   (b) Define :

   (i) Four dimensional radius vector

   (ii) Four vector A^r

   (iii) Light-like interval

   (iv) World line. 4

UNIT—IV

8. (a) Deduce Einstein's mass-energy equivalence relation. 6

   (b) A particle is given a kinetic energy equal to n times it's rest energy m_0c^2. What are :

   (i) its speed and

   (ii) momentum ? 4

9. (a) Prove that the mass of a moving particle with velocity u is $m = \frac{m_0}{\sqrt{1 - u^2/c^2}}$, where m_0 is the mass of the particle when it is at rest. 6

   (b) Show that four velocity and four acceleration are mutually orthogonal. 4

UNIT—V

10. (a) Obtain the transformations for electric and magnetic field strengths. 6

(b) Prove that the energy momentum tensor of electromagnetic field is trace free. 4

11. (a) Show that the Lorentz force acting on a particle of charge e is given by $\overline{F}_L = e\left(\overline{E} + \frac{1}{c}\overline{u} \times \overline{H}\right)$. 6

(b) Show that the Hamiltonian for a charged particle moving in an electromagnetic fields is :

$$H = \left\{ m_0^2 c^4 + c^2 \left(P - \frac{e}{c} A \right)^2 \right\}^{1/2} + e\phi.$$ 4