AU–155

B.Sc. (Part-III) Semester-VI Examination

MATHEMATICS (OLD) UPTO WINTER-2018

(Special Theory of Relativity)

Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) "All inertial frames are equivalent." This statement is called : 1

(a) Principle of relativity (b) Special relativity

(c) Galilean principle of relativity (d) None of these

(ii) In special relativity, the simultaneity is : 1

(a) absolute (b) relative

(c) constant (d) None of these

(iii) The relativistic addition law for velocity is : 1

(a) $u' = u - v$ (b) $u' = c$

(c) $u' = \dfrac{u - v}{1 - \dfrac{uv}{c^2}}$ (d) None of these

(iv) The conclusion 'moving clock go slow' is : 1

(a) Time dilation (b) Length contraction

(c) Lorentz contraction (d) None of these

(v) At each contraction, the order of the tensor is reduced by : 1

(a) One (b) Two

(c) Three (d) Four

(vi) $\delta^r_s A^s = ...$ 1

(a) A^s (b) A_s

(c) A^r (d) A_r

(vii) The force acting on the particle is parallel to the acceleration when the velocity of the particle is : 1

(a) Parallel to its acceleration

(b) Perpendicular to its acceleration

(c) Either parallel or perpendicular to its acceleration

(d) Neither parallel nor perpendicular to its acceleration

(viii) The four velocity of the particle is a unit : 1

(a) Null like vector (b) Light like vector

(c) Space like vector (d) Time like vector

(ix) The gauge invariance implies that the scalar potential is defined to within the : 1

(a) Time derivative of the function f (b) Gradient of an arbitrary function f

(c) Curl of the function f (d) None of these

(x) The energy momentum tensor of electromagnetic field is : 1

(a) Antisymmetric (b) Without trace free

(c) Zero (d) Symmetric

UNIT—I

2. (a) Prove that $\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is invariant under Lorentz transformation. 6

(b) Show that Simultaneity is relative but not absolute in special relativity. 4

3. (p) Derive the Lorentz transformation equations. 6

(q) Prove that Newton's fundamental equations of motion are invariant under the Galilean transformations. 4

UNIT—II

4. (a) Explain a short note on Length contraction. 5

(b) If $\vec{u}$ and $\vec{u}'$ be the velocities of a particle in two inertial system s and s' respectively where s' is moving with velocity v relative to s along the xx' axis, then show that :

$$\tan\theta' = \frac{\sin\theta\left(1-\frac{v^2}{c^2}\right)^{1/2}}{\left(\cos\theta-\frac{u}{v}\right)}. \qquad 5$$

5. (p) Derive the transformation of Lorentz contraction factor $\left(1-\frac{u^2}{c^2}\right)^{1/2}$. 6

(q) An observer moving along the x-axis of s with velocity v observes a body of proper volume v_o moving with velocity u along the x-axis of s. Show that the observer measures the volume to be equal to :

$$v_o\sqrt{\frac{(c^2-v^2)(c^2-u^2)}{(c^2-uv)^2}}. \qquad 4$$

UNIT—III

6. (a) Show that the quantity $s^2 = -(x')^2 - (x^2)^2 - (x^3)^2 + (x^4)^2$ is invariant under the Lorentz transformations. 4

(b) Deduce the transformations for an anti symmetric four tensor T'^{12}, T'^{13} and T'^{24}. 2+2+2

7. (p) Define :

(i) Covariant tensor of rank two

(ii) Contravariant tensor of rank two

(iii) Mixed tensor of rank two

(iv) Proper time

(v) Time like. 1+1+1+1+1

(q) Show that the metric $ds^2 = -(dx')^2 - (dx^2)^2 - (dx^3)^2 + (dx^4)^2$. 5

UNIT—IV

8. (a) Derive mass energy equivalence relation $E = mc^2$. 6

(b) Show that transformation equation of mass under Lorentz transformation is :

$$m = \alpha\left(1 + \frac{u'_x v}{c^2}\right)m'.$$ 4

9. (p) Deduce the transformations of the components of four force f^i in the form :

$$f^i = \left(\frac{\vec{f}}{c\cdot\sqrt{1-u^2/c^2}}, \frac{\vec{f}\cdot\vec{u}}{c^2\sqrt{1-u^2/c^2}}\right).$$ 5

(q) Define four velocity and four acceleration. Show that four velocity and four acceleration are mutually orthogonal. 2+3

UNIT—V

10. (a) Prove that the Lagrangian for a charged particle in electromagnetic field is :

$$L = -m_0 c^2\sqrt{1-\frac{u^2}{c^2}} + \frac{e}{c}\vec{A}\cdot\vec{u} - e\phi.$$ 5

(b) Suppose that an electromagnetic field is purely magnetic in an inertial frame S. Describe the field in inertial frame S'. 5

11. (p) Prove that the Lorentz force acting on a charge e is :

$$\vec{F}_L = e\vec{E} + \frac{e}{c}\vec{u}\times\vec{H}.$$ 5

(q) State the Maxwell's equations of electromagnetic theory in vacuum and write its equations in component form. 5