AT–400

B.Sc. (Part-III) Semester-VI Examination

6S-MATHEMATICS

Special Theory of Relativity

Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it at once only.

(2) Solve **ONE** question from each Unit.

1. Choose the correct alternatives :

(1) If $ds^2 = 0$, then the interval 'ds' is said to be : 1

(a) Light like (b) Space like

(c) Time like (d) None of these

(2) If an electromagnetic field is purely electric in an inertial frames, then the field in s' is : 1

(a) Only electric (b) Only magnetic

(c) Electric as well as magnetic (d) None of these

(3) If ϕ is a scalar potential and $\overline{A}$ is the vector potential, then the electric field is given by : 1

(a) $\overline{E} = \text{grad}\,\phi - \frac{1}{c}\frac{\partial \overline{A}}{\partial t}$ (b) $\overline{E} = \text{grad}\,\phi + \frac{1}{c}\frac{\partial \overline{A}}{\partial t}$

(c) $\overline{E} = -\text{grad}\,\phi - \frac{1}{c}\frac{\partial \overline{A}}{\partial t}$ (d) $\overline{E} = -\text{grad}\,\phi + \frac{1}{c}\frac{\partial \overline{A}}{\partial t}$

(4) The electric field strength $\overline{E}$ and the magnetic field strength $\overline{H}$ are invariant under : 1

(a) Galilean Transformations (b) Lorentz Transformations

(c) Gudge Transformations (d) None of these

(5) The time recorded by a clock moving with a body is called as : 1

(a) Absolute time (b) Proper time

(c) Improper time (d) None of these

(6) Length contraction means : 1

(a) Moving rod measures longer (b) Rest rod measures longer

(c) Moving rod measures shorter (d) Rest rod measures shorter

(7) Inertial system means the reference system where : 1

(a) Newton's first law of motion is valid

(b) Newton's second law of motion is valid

(c) Newton's third law of motion is valid

(d) None of these

(8) Four velocity of a particle is a : 1

(a) Unit space like vector (b) Unit time like vector

(c) Unit light like vector (d) None of these

(9) The simultaneity in special relativity is : 1

(a) Constant (b) Relative

(c) Absolute (d) None of these

(10) The interval ds is said to be space-like if : 1

(a) $ds^2 = 0$ (b) $ds^2 > 0$

(c) $ds^2 < 0$ (d) None of these

UNIT—I

2. (a) Prove that in an inertial frame a body without influence of any forces moves in a straight line with constant velocity. 3

(b) Show that Newton's kinematical equations of motion are invariant under Galilean transformations. 4

(c) Show that $x^2 + y^2 + z^2 - c^2t^2$ is Lorentz invariant. 3

3. (p) Show that Lorentz transformation forms a group with respect to multiplication. 4

(q) Prove that $\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is invariant under special Lorentz transformations. 4

(r) What are the postulates of special relativity ? 2

UNIT—II

4. (a) Show that in nature no signal can move with a velocity greater than the velocity of light relative to any inertial system. 5

(b) Show that the velocities u and u' measured in two inertial systems and s' are related by :

$$\sqrt{1-\frac{u^2}{c^2}} = \frac{\sqrt{1-\frac{u'^2}{c^2}}\cdot\sqrt{1-\frac{v^2}{c^2}}}{\left(1+\frac{u'_x v}{c^2}\right)},$$

where s' is moving with velocity v relative to s along XX' axis. 5

5. (p) Obtain the transformations for the acceleration of a particle under special Lorentz transformations. 4

(q) Explain :

(i) Time Dilation

(ii) Length contraction. 3+3

UNIT—III

6. (a) Define four vector. Show that :

$A^1 = -A_1$, $A^2 = -A_2$, $A^3 = -A_3$, $A^4 = A_4$. 4

(b) Prove that there exists an inertial system s' in which the two events occurs at one and the same time if the interval between two events is time like. 4

(c) What do you mean by covariant and contravariant tensor of rank two ? 2

7. (p) Define : Proper time. Show that the proper time of a moving object is always less than the corresponding interval in the rest system. 4

(q) Obtain the transformation of the components T'^{11} and T'^{12}. 4

(r) What are world points and world line ? 2

UNIT—IV

8. (a) Define four velocity. Prove that the four velocity in component form can be expressed as :

$$u^i = \left(\frac{\bar{u}}{c\sqrt{1-\frac{u^2}{c^2}}}, \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \right),$$

where $\bar{u} = (u_x, u_y, u_z) =$ velocity of the particle. 1+3

(b) Show that the quantity $p^2 - \frac{E^2}{c^2}$ is an invariant whose numerical value is $-m_0^2c^2$. 3

(c) Prove that four velocity and four acceleration are mutually orthogonal. 3

9. (p) Obtain the mass energy equivalence relation $E = mc^2$, where m is the relativistic mass of the particle. 4

(q) Prove that $E = c\sqrt{p^2 + m_0^2c^2}$ and $\frac{dE}{dp} = u$. 3

(r) Define four force. Show that the four force and the four velocity are orthogonal to each other. 1+2

UNIT—V

10. (a) Define current four vector. Show that $c^2\rho^2 - J^2$ is invariant and its value is $p_0^2.c^2$. 1+4

(b) Prove that the set of Maxwell's equations $\text{div}\,\bar{H} = 0$ and $\text{curl}\,\bar{E} = -\frac{1}{c}\frac{\partial \bar{H}}{\partial t}$ can be written as $\frac{\partial F_{ij}}{\partial x^k} + \frac{\partial F_{jk}}{\partial x^i} + \frac{\partial F_{ki}}{\partial x^j} = 0$, where F_{ij} is the electromagnetic field tensor. 5

11. (p) Define electromagnetic field tensor F_{ij}. Express the components of F_{ij} in terms of the electric and magnetic field strengths. 1+4

(q) Obtain the transformations for electric and magnetic field strengths. 5