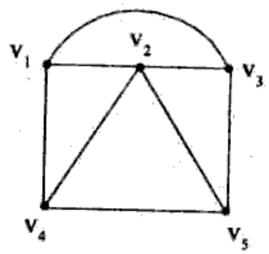
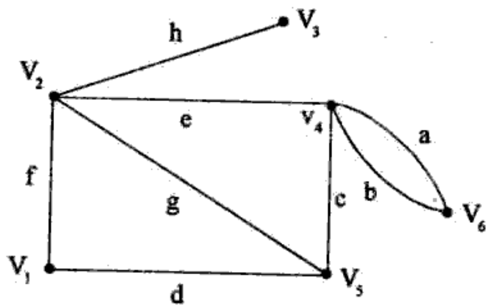
(b) Define and find adjacency matrix for the following graph : 5



11. (p) Find the path matrix $P(V_3.V_4)$ and circuit matrix B(G) for the following graph G :



Graph G

5

(q) If B is a circuit matrix of a connected graph G with e edges and n vertices, then rank of $B = e - n + 1$. Prove this. 5

AP–512

B.Sc. Part–III (Semester–VI) Examination

6S : MATHEMATICS—XII

(Graph Theory)

(Optional)

Time—Three Hours] [Maximum Marks—60

Note :— (1) Question No. **1** is compulsory and attempt at once.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternatives : 1 mark each

(i) In a simple graph, if every two distinct vertices in it are adjacent, then it is said to be :

(a) Euler graph

(b) Hamiltonian graph

(c) A connected graph

(d) A complete graph

(ii) A graph G is called null graph if degree of each vertex is :

(a) Odd (b) Even

(c) Zero (d) One

(iii) "There are n^{n-2} labelled trees with n vertices $(n \geq 2)$" is the statement of :

(a) Cayley formula

(b) Hamiltonian formula

(c) Euler formula

(d) Kuratowski formula

(iv) A tree in which one vertex is distinguished from all others is called a :

(a) Binary tree (b) Rooted tree

(c) Free tree (d) Spanning tree

(v) Let T be a spanning tree of a connected graph G. Adding any one chord to T will create exactly :

(a) One circuit (b) One cut-set

(c) Two circuits (d) Two cut-sets

(vi) A connected graph is said to be separable if its vertex connectivity is :

(a) 0 (b) 1

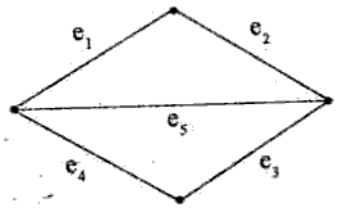
(c) 2 (d) 3

(vii) Let graph G be connected with 4 vertices and 5 edges then nullity (μ) =.

(a) 9 (b) 5

(c) 4 (d) 2

(b) For the given graph G, find W_G, W_s, W_r, $W_s \cap W_r$ and $W_s \cup W_r$. 5



Graph G

9. (p) Show that the set W_s of all cut-set vectors including zero vector, in W_G forms a subspace of W_G. 5

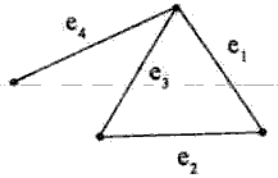
(q) Find :

(i) $W_r \cap W_s$

(ii) $W_r \cup W_s$

(iii) $W_r \vee W_s$

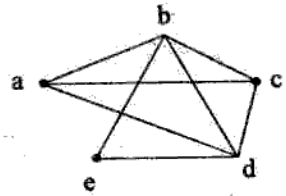
for the following graph : 5



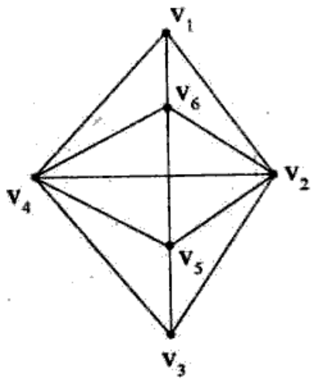
UNIT–V

10. (a) Define rank of an incidence matrix. If A(G) be an incidence matrix of a connected graph G with n vertices then prove that rank of A(G) is n – 1. 5

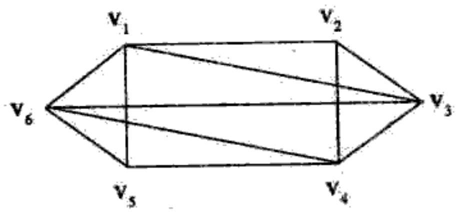
(ii)



(iii)



(iv)



UNIT–IV

8. (a) Prove that the circuit subspace W_r and the cutset subspace W_s are orthogonal to each other in vector space of a graph. 5

(viii) Two subspaces W_1 and W_2 are said to be orthogonal to each other iff for all $X \in W_1$ and $Y \in W_2$, $X \cdot Y =$.

(a) $X - Y$ (b) $X + Y$

(c) 0 (d) 1

(ix) In a graph without loops, the entries along principal diagonal of Adjacency matrix are all :

(a) Zeros (b) 1's

(c) Purely imaginary (d) Complex numbers

(x) Reduced incidence matrix of a graph is non-singular iff the graph is :

(a) A circuit (b) A tree

(c) A cutset (d) A regular

UNIT–I

2. (a) If a connected graph G is decomposed into circuits iff G is an Euler graph, prove this. 5

(b) Define isomorphism in graph. Are the two graphs given below isomorphic to each other ? Why ?

Graph G_1

Graph G_2 5

3. (p) Define simple graph. Prove that a simple graph with n vertices and k-components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. 5

(q) Define graph. Draw graphs of the following chemical compounds :

(i) CH_4

(ii) C_2H_6

(iii) C_6H_6

(iv) N_2O_3 5

UNIT–II

4. (a) Define a binary tree. Prove that :

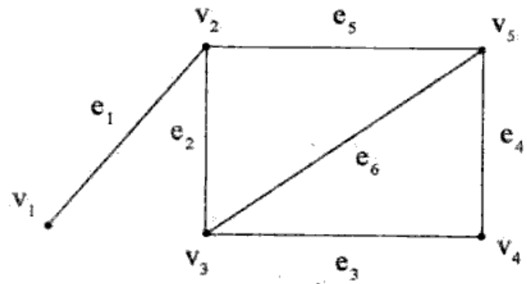
(i) Every binary tree has an odd number of vertices.

(ii) There are $\frac{n+1}{2}$ pendant vertices in any binary tree with n vertices. 5

(b) Prove that a graph G with n vertices and $(n-1)$ edges and having no circuit is connected. 5

5. (p) Prove that a graph T is a tree if and only if there is one and only one path between every pair of vertices in T. 5

(q) Define a spanning tree of a connected graph. Sketch all spanning trees of the following graph :



5

UNIT–III

6. (a) Prove that ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets. 5

(b) Using geometric arguments prove that the graph $K_{3,3}$ is non-planar. 5

7. (p) Define the region and prove that if G is a planar graph with n vertices, e edges and f region then

$$n - e + f = 2$$ 5

(q) Define planar graph. Show that each graph is planar by redrawing it such that no edge cross : 5

(i)