**B.Sc. (Part–III) Semester–VI Examination**

**MATHEMATICS**

**(Special Theory of Relativity)**

**Paper—XII**

Time—Three Hours] [Maximum Marks—60

**Note :**— (1) Question No. **1** is compulsory.
(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternative :

(i) The special Lorentz transformations will reduce to simple Galilean transformations when :

(a) $V << C$

(b) $V >> C$

(c) $V = C$

(d) None of these 1

(ii) The reference system is said to be an inertial system if :

(a) Newton's second law of motion valid

(b) Newton's first law of motion valid

(c) Newton's third law of motion valid

(d) None of the above 1

(iii) The interval ds is said to be time-like if :

(a) $ds^2 < 0$

(b) $ds^2 = 0$

(c) $ds^2 > 0$

(d) None of these 1

(iv) Time recorded by a clock moving with a body is known as :

(a) Proper time

(b) Improper time

(c) Fixed time

(d) None of these 1

(v) The numerical value of the quantity $p^2 - E^2/c^2$ is :

(a) $m_0^2 c^2$

(b) $-m_0^2 c^2$

(c) $m_0^2 c$

(d) $m_0 c^2$ 1

(vi) Four velocity of a particle is defined as :

(a) $u = \frac{dx^i}{ds}$

(b) $u^i = \frac{dx_i}{ds}$

(c) $u^i = \frac{dx^i}{ds^i}$

(d) $u^i = \frac{dx^i}{ds}$ 1



(b) Prove that the four velocity in component form can be expressed as $u^i = \left(\frac{u}{c\sqrt{1-u^2/c^2}}, \frac{1}{\sqrt{1-u^2/c^2}}\right)$. 4

9. (p) Show that the quantity $p^2 - E^2/c^2$ is an invariant and whose numerical value is $-m_0^2 c^2$. 4

(q) Prove that the four velocity and four acceleration are mutually orthogonal. 3

(r) Obtain the transformation of mass of a particle. 3

**UNIT—V**

10. (a) Suppose that an electromagnetic field is purely electric in an inertial frame S. Describe the field in an inertial frame S'. 5

(b) Prove that the set of Maxwell's equations $\text{div } \bar{E} = 0$; $\text{curl } \bar{H} = \frac{1}{c}\frac{\partial \bar{E}}{\partial t}$ can be expressed as $\frac{\partial F^{ik}}{\partial x^k} = 0$. 5

11. (p) Prove that electromagnetic field strength $\bar{E}$ and $\bar{H}$ remain invariant under Gauge transformations. 4

(q) Obtain the transformations for electric and magnetic field strengths. 6

## UNIT—III

6. (a) Define :

(i) Four vectors

(ii) Time like vector

(iii) Space like vector

(iv) Null vector. 4

(b) Obtain the transformation of the components $T'^{12}$ and $T'^{41}$. 4

(c) Show that $A^1 = -A_1$, $A^2 = -A_2$, $A^3 = -A_3$, $A^4 = A_4$. 2

7. (p) Define contravariant tensor of order two and hence obtain the transformation of the components $T'^{11}$ and $T'^{14}$. 5

(q) Prove that there exists an inertial system S′ in which the two events occur at one and the same point if the interval between two events is time like and also show that such events cannot occur simultaneously in any inertial system. 5

## UNIT—IV

8. (a) Prove that the mass of a moving particle with velocity u is $m = \frac{m_0}{\sqrt{1-u^2/c^2}}$; where $m_0$ is the mass of the particle when it is at rest. 6



(vii) A set of quantities $T_i$ at a point $P \in V_4$ are called covariant tensor of order one if they obey the transformation law given by :

(a) $T_i' = \frac{\partial x^j}{\partial x'^i} T_j$

(b) $T_i' = \frac{\partial x_j}{\partial x'^i} T_j$

(c) $T_i' = \frac{\partial x^j}{\partial x_i'} T_j$

(d) $T_i' = \frac{\partial x^j}{\partial x'^i} T^j$ 1

(viii) Four forces $f^i$ are defined as :

(a) $f^i = \frac{dp^i}{ds}$

(b) $f^i = \frac{dp^i}{dt}$

(c) $f^i = \frac{dp_i}{ds}$

(d) None of these 1

(ix) Which of the following are Gauge transformations ?

(a) $A' = A - \nabla f,\ \phi' = \phi - \frac{1}{c}\frac{\partial f}{\partial t}$

(b) $A' = A + \nabla f,\ \phi' = \phi - \frac{1}{c}\frac{\partial f}{\partial t}$

(c) $A' = A + \nabla f,\ \phi' = \phi + \frac{1}{c}\frac{\partial f}{\partial t}$

(d) $A' = A - \nabla f,\ \phi' - \frac{\partial f}{\partial t}$ 1

(x) The electromagnetic field tensor $F_{ij}$ is said to be antisymmetric if :

(a) $F_{ij} = F_{ji}$
(b) $F_{ij} = -F_{ji}$
(c) $F_{ij} = -F_{ij}$
(d) $F_{ij} = F^{ij}$ 1

## UNIT—I

2. (a) Show that the Lorentz transformations form a group with respect to composition. 5

(b) The space-time coordinates of two events as measured in a frame S are as follows :

Event 1 : $x_1 = x_0,\ t_1 = \frac{x_0}{c},\ y_1 = 0,\ z_1 = 0$

Event 2 : $x_2 = 2x_0,\ t_2 = \frac{x_0}{2c},\ y_2 = 0,\ z_2 = 0.$

(i) Find the velocity of an inertial frame with respect to S where these events occur at the same time.

(ii) What is the value of t at which both events occur in the new frame ? 5



3. (p) State and prove special Lorentz transformations. 6

(q) Prove that in an inertial frame particle moves in a straight line with constant velocity. Also prove that equation of motion of a particle have the same form in all inertial frame. 4

## UNIT—II

4. (a) Deduce the transformations of particle velocities, and hence obtain Einstein's velocity addition law. 5

(b) Obtain the transformation of the Lorentz contraction factor $(1 - u^2/c^2)^{1/2}$. 5

5. (p) Let u and u' be the velocities of a particle in two inertial system S and S' respectively, where S' is moving with velocity $v$ relative to S along XX' axis.

Show that $\tan\theta' = \dfrac{\sin\theta\left(1 - v^2/c^2\right)^{1/2}}{\cos\theta - v/u}$ and

$$u'^2 = \frac{u^2\left\{1 - \frac{2v}{u}\cos\theta + \left(\frac{v}{u}\right)^2 - \left(\frac{v}{c}\right)^2\sin^2\theta\right\}}{\left(1 - \frac{uv}{c^2}\cos\theta\right)^2}$$

where $\theta$ and $\theta'$ are the angles between the x-axis and the vectors u and u' respectively. 5

(q) Prove that moving rod measures shorter in lengths. 5