M.E. First Semester (Electronics & Tele.) (Full Time) (C.G.S.- New)

13332 : Random Processes : 1 ENTC 2

P. Pages : 3 | AU - 3458

Time : Three Hours | Max. Marks : 80

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Notes :
1. Assume suitable data wherever necessary.
2. Illustrate your answer necessary with the help of neat sketches.
3. Use of pen Blue/Black ink/refill only for writing the answer book.

SECTION – A

1. a) A binary communication transmitter sends data as one of two types of signals denoted by 0 or 1. Due to noise, some times a transmitted 1 is received as 0 and vice versa. If the probability that a transmitted 0 is correctly received as 0 is 0.9 and the probability that a transmitted a 1 is correctly received as 1 is 0.8 and if the probability of transmitting 0 is 0.45. Find the probability that (i) a 1 is received, (ii) a 0 received (iii) a 1 was transmitted given that 1 was received, (iv) a 0 was transmitted given that a 0 was received (v) the error has occurred. **7**

   b) Explain expectation, moments and variance with an example. **7**

OR

2. a) For exponential PDF $f(x) = b e^{-a|x|}$ **7**
   1) Find relation between a and b.
   2) CDF
   3) Find $P(1 \le x \le 2)$.

   b) The CDF of a continuous random variable X is given by **7**

$$Fx(x) = \begin{cases} 0, & x < 0 \\ x^2, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases}$$

   Find the PDF. Draw the graphs of both PDF and CDF. Also find $P\left(\frac{1}{2} \le x \le \frac{4}{5}\right)$.

3. a) Common probabilities for uniform distribution for a RV $X \sim u\,(4, 6)$. Find its mean $\overline{x} = \int_{-\infty}^{\infty} x\, f(x)\, dx$ and standard deviation $6 = \sqrt{\int_{-\infty}^{\infty} (x-\overline{x})^2 f(x) dx}$ and the following probabilities. **7**

   a) $P(X-\overline{x} < 6)$ b) $P(X-\overline{x} < 26)$

   c) $P[|X-\overline{x}| < 6]$ d) $P[|X-\overline{x}| < 26]$

   b) X is a normal variate with mean 30 and standard deviation 5. Find the probabilities that **6**

   i) $26 \le x \le 40$ ii) $x \ge 45$ iii) $|X-30| \ge 5$

OR

4. a) A transmission channel has a per digit error probability P = 0.01 calculate the probability of more than 1 error in 10 received digits using (i) Binomial Distribution (ii) Poisson distribution. **6**

b) If $X_1, X_2, -----X_{50}$ are independent and identically distributed random variable each having Poisson distribution with parameter m = 0.03 and if $S_n = X_1 + X_2 + ----+X_{50}$. Find $P(S_n \geq 3)$ using central limit theorem. **7**

5. a) The random variable $X_1, X_2, -----X_n$ have the Joint PDF as : **7**

$$f_{X_1, X_2 ---, X_n}(X_1, X_2 ---, X_n) = \begin{cases} 1 ; & 0 \leq x_i \leq i \\ & i = 1, 2, ---n \\ 0 ; & \text{otherwise} \end{cases}$$

i) What is the Joint CDF : $F_{X_1, X_2 ---, X_n}(X_1, X_2 ---, X_n)$?

ii) For n = 3, what is the probability that mini $X_i \leq 3/4$?

b) Explain mean and variance of weighted sum of random variable. **6**

OR

6. a) We are given that X Gaussian random vector with **7**

$$\mu_X = \begin{bmatrix} 4 \\ 8 \\ 6 \end{bmatrix} \text{ and } C_X = \begin{bmatrix} 4 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 4 \end{bmatrix}$$

Let $y = AX + b$, where

$$A = \begin{bmatrix} 1 & 1/2 & 2/3 \\ 1 & -1/2 & 2/3 \end{bmatrix} \text{ and } b = [-4 \quad -4]^T$$

Calculate :

i) The expected value, μ_y

ii) The covariance, C_y

iii) The probability that $-1 \leq y_2 \leq 1$.

b) Explain covariance & covariance matrix with example. **6**

SECTION – B

7. a) If $[X(t)]$ is a Gaussian process with mean $\mu(t) = 5$ and $C(t_1, t_2) = 4e^{-0.2|t_1 - t_2|}$ find the probability that **7**

i) $X(8) \leq 3$ ii) $|X(8) - X(3)| \leq 3$

b) Explain in detail **6**

1) Wiener process.

2) Stationery process.

OR

8. a) Explain in detail with necessary mathematical framework the standard Brownian motion. Hence Explain the diffusion process and the martingale process associated with Brownian process. **7**

b) Explain following term in detail. **6**
   1) White noise
   2) Random walk.

9. a) If a random process is given by $X(t) = A\cos(2\pi t + y)$ where, A is a constant and y is discrete random variable with $P(y=0)=1/2$ and $P(y=\pi/2)=1/2$, find the mean $M_X(t)$ and the autocorrelation function $R_X, X(t_1, t_2)$. Also find the values of $m_X(1)$ and $R_X(0,1)$. **7**

b) Define : **6**
   i) Autocorrelation function (ACF).
   ii) Crosscorrelation function (CCF).

OR

10. a) Explain cross-covariance and cross-correlation coefficient between two random processes X(t) and Y(t). **7**

b) Find the autocorrelation function of the periodic random process given by $X(t) = A\sin wt$ **6**

11. a) Given the power spectral density of a continuous process as $s(w) = \dfrac{w^2+9}{w^4+5w^2+4}$ find the mean square value of the process. **7**

b) Explain the power spectrum of sum of two Random process. **6**

OR

12. a) A random process has auto-correlation function given by : **7**
$R_{X(\tau)} = 7e^{-4|\tau|} - 2e^{-2|\tau|}\cos(3\pi\tau) + 2\cos(4\pi\tau)$
find the power spectrum.

b) Define cross power spectrum. Explain power spectrum estimation in Laplace domain. **6**

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