M.E. First Semester (Mechanical Engineering (Thermal Engg.)) (New-CGS)

13505 : Advanced Mathematics - III : 1 MTE 1

P. Pages : 2 | **AU - 3359**

Time : Three Hours | Max. Marks : 80

Notes : 1. All question carry equal marks.
2. Answer **two** question from Section A and **two** question from Section B.
3. Assume suitable data wherever necessary.
4. Use of calculator is permitted.
5. Use of pen Blue/Black ink/refill only for writing book.

SECTION - A

1. a) Solve **6**

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cdot \cos 2y .$$

b) Solve **7**

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y .$$

c) Solve **7**

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \cdot \cos x .$$

2. a) Solve **6**

$$\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} - 4\frac{\partial^2 z}{\partial y^2} = x + \sin y$$

b) Solve by the method of separation of variables. **8**

$$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$$

and $u = e^{-5y}$, when x=0.

c) Solve **6**

$$\left(D^3 - 6D^2 D' + 12DD'^2 - 8D'^3\right) z = 0 .$$

3. a) Solve the equation by method of separation of variables- **7**

$$\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cdot \cos x ,$$

given that u=0 when t=0, $\frac{\partial u}{\partial t} = 0$ when x=0.

b) A rectangular plate with insulated surface is 10cm. wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature of the short edge y=0 is given by **13**

$u = 20x$ for $0 < x \leq 5$

and $u = 20(10-x)$ for $5 \leq x \leq 10$ and the two long edges $x=0$; $x=10$ as well as the other short edge are kept at 0°C prove that the temperature u at any point (x, y) is given by.

$$u = \frac{800}{\pi^2} \sum_{1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^2} \cdot \sin \frac{(2n-1)\pi x}{10} e^{-\frac{(2n-1)\pi y}{10}}$$

SECTION - B

4. a) Fit a straight line to the following data. Also find the expected production in the year 1956. **6**

Year (x)	Production (y) (in matric tones)
1911	8
1921	10
1931	12
1941	10
1951	16

b) Find the coefficient of correlation for the data. **7**

x:	1	2	3	4	5	6	7	8	9	10
y:	10	12	16	28	25	36	41	49	40	50

c) In the class of 100 students there are 40 girls and 60 boys 10 of the girl and 20 of the boys are failed in mathematics. The roll numbers selected at random is found to be that of a student who has failed in mathematics. What is the probability that of girl? **7**

5. a) A function f(x) is given by the following table. Find f (0.2) by a suitable formula. **10**

x:	0	1	2	3	4	5	6
f(x):	176	185	194	203	212	220	229

b) Using Lagrange's formula, find the form of the function f(x) given that: **10**

x:	0	2	3	6
f(x):	659	705	729	804

6. a) Use trapezoidal rule to evaluate $\int_4^8 \frac{dx}{x}$, using four equal sub-intervals. **6**

b) Given that $\frac{dy}{dx} = 2 + \sqrt{xy}$ and y (1) = 1. Find y (2) in steps of 0.2 using Euler's method. **7**

c) Use Runge-Kutta method to compute y (0.2) in steps of 0.1; if $\frac{dy}{dx} = x + y^2$; given that y = 1 when x=0. **7**

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