

M.Sc. (Part—I) Semester—I (C.B.C.S. Scheme) Examination
1 SCA 1 : STATISTICS
(Elementary Probability and Distribution Theory)
Paper—I

Time : Three Hours]

[Maximum Marks : 80

Note :— Solve (A) or (B) from each question.

1. (A) (i) Explain the concepts :
- (a) Random variable
 - (b) Probability mass function
 - (c) Probability density function
 - (d) Frequency approach of probability.
- (ii) Define conditional expectation and conditional variance. Show that :
- (a) $E(X) = E[E(X/Y)]$
 - (b) $V(X) = E[V(X/Y)] + V[E(X/Y)]$
- where $E(X^2) < \infty$.

8+8

OR

- (B) (i) State and prove Boole's inequality.
- (ii) The joint p.d.f. of two continuous r.v's X and Y is :

$$f(x, y) = \left(\frac{3x + y}{4} \right) e^{-x-y}, x > 0, y > 0$$

Find the marginal densities of X and Y and the conditional densities $f(x/y)$ and $f(y/x)$.

8+8

2. (A) (i) State the p.d.f. of Poisson distribution. Obtain its mode. Also derive the additivity property of the Poisson distribution.
- (ii) Give the probability mass function of the negative binomial distribution. Obtain an expression for its M.G.F. Hence find mean and variance.

8+8

OR

- (B) (i) Explain uniform distribution. Find its mean and variance.
- (ii) Discuss hypergeometric distribution. Obtain the mean and variance of hypergeometric distribution.
3. (A) (i) State the p.d.f. of Laplace distribution and obtain the expression for r^{th} raw moment. Hence obtain its mean and variance.
- (ii) Define beta distribution of first kind. Obtain its mean and variance.

8+8

OR

(B) (i) State the p.d.f. of exponential distribution. Obtain its mean and variance.

(ii) If x and y are two independent standard normal variates then find the p.d.f. of $\frac{x}{y}$ and identify it. 8+8

4. (A) (i) Show that if a continuous r.v. X follows $F(n_1, n_2)$ d.f. then $(1/X)$ also follows F distribution. Also obtain the mode of the F distribution.

(ii) State and prove Liapunov inequality. 8+8

OR

(B) (i) If χ_1^2 and χ_2^2 are two independent Chi-square variates with n_1 and n_2 d.f., obtain the p.d.f. of $\frac{\chi_1^2/n_1}{\chi_2^2/n_2}$. Also obtain the mode of the distribution.

(ii) Define Students t -statistic. Give any two applications of t -distribution stating the assumptions made. 8+8

5. (A) (i) What is truncation ? Find mean and variance of Binomial distribution truncated at $X = 0$.

(ii) Define order statistics. State their joint distribution function. Derive the marginal distribution of the lowest order statistic. 8+8

OR

(B) (i) Explain the concept of compound and mixture distributions and their importance. Illustrate by giving suitable examples.

(ii) Let $X_1, X_2, \dots, X_n$ be a random sample with p.d.f. given by :

$$f(x) = 1 - x, \quad 0 < x < 1 \\ = 0, \quad \text{otherwise}$$

Find the p.d.f. and variance of $X_{(1)}$ where $X_{(1)} = \min(X_1, \dots, X_n)$. 8+8