AQ – 803

First Semester M.A./M.Sc. (Part – I) (CBCS)
Examination
(Old Course)

MATHEMATICS

Paper – I MTH 6 (106)
(Advanced Discrete Mathematics – I)

P. Pages : 5

Time : Three Hours] [Max. Marks : 80

Note : Solve **One** question from each unit.

UNIT I

1. (a) Show that (without truth table)

   (i) $P \to (Q \to R) \Leftrightarrow P \to (\sim Q \vee R) \Leftrightarrow (P \wedge Q) \to R$

   (ii) $(\sim P \wedge (\sim Q \wedge R)) \vee ((Q \wedge R) \vee (P \wedge R)) \Leftrightarrow R$ 8

   (b) Define and symbolise.

   (i) Predicates.

   (ii) Quantifiers, Also state rules US, ES, EG and UG 8

2. (c) Write the rules of generalization and show $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$ and is the converse true ? Explain. 8

(d) Write the rules of inference and show that RVS follows logically from the premises CVD, $(CVD) \rightarrow \neg H$, $\neg H \rightarrow (A \wedge \neg B)$ and $(A \wedge \neg B) \rightarrow (RVS)$. 8

UNIT II

3. (a) Show that for any commutative monoid (M, *) the set of idempotent elements of M forms a submonoid. 8

(b) State and prove fundamental theorem of semigroup homomorphism. 8

4. (c) Let X be a set containing n elements, let X^* denote the free semigroup generated by X, and let $\langle S, \oplus \rangle$ be any other semi-group generated by any n generators, then prove that there exists a homomorphism $g: x^* \rightarrow S$ 8

(d) Let $\langle S, * \rangle$ and $\langle T, \Delta \rangle$ be two semigroups and g be a semigroup homomorphism from $\langle S, * \rangle$ to $\langle T, \Delta \rangle$. Corresponding to the homomorphism g, there exists a congruence relation R on $\langle S, * \rangle$ defined by $x R y$ iff $g(x) = g(y)$ for $x, y \in S$, prove this. 8

UNIT III

5. (a) Let $\langle L, \leq \rangle$ be a lattice, for any $a, b, c \in L$. Then prove the following inequalities

$$a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$$

$$a * (b \oplus c) \geq (a * b) \oplus (a * c)$$ 8

(b) Define chain and prove that every chain is a distributive lattice. 8

6. (c) Show that De-Morgan's law given by $(a*b)' = a' \oplus b'$ and $(a \oplus b)' = a' * b'$ holds in complemented distributive lattice. 8

(d) Define Distributive lattice and show that the direct product of any two distributive lattice is distributive. 8

UNIT IV

7. (a) In any Boolean algebra, show that

(i) $(a+b)(a'+c) = ac + a'b = ac + a'b + bc$

(ii) $a = 0 \Leftrightarrow ab' + a'b = b$ 8

(b) Prove the following Boolean identities.

(i) $(a*b*c)\oplus(a*b)=a*b$

(ii) $a\oplus(a'*b)=a\oplus b$ 8

8. (c) Define Boolean homomorphism. Show that a lattice homomorphism on a Boolean algebra which preserve 0 and 1 is a Boolean homomorphism. 8

(d) If p, q, r, a, b are elements of Boolean algebra then prove that

(i) $pqr + pqr' + pq'r + p'qr = pq + qr + rp$

(ii) $ab + ab' + a'b + a'b' = 1$ 8

UNIT V

9. (a) Use k-map to minimise the Boolean expression.

(i) $f(a, b, c, d) = \Sigma(0, 1, 2, 3, 13, 15)$

(ii) $f(a, b, c, d) = \Sigma(0, 5, 7, 8, 12, 14)$ 8

(b) Obtain product of sums of cannonical form of boolean expression.

(i) $x_1 \wedge x_2$

(ii) $x_1 \vee x_2$ 8

10. (c) Obtain sum of product cannonical form in three variables. x_1, x_2, x_3 of

(i) $x_2 \vee x_3'$

(ii) $(x_1 \vee x_2)' \vee (x'_1 \wedge x_3)$ 8

(d) Obtain the values of the Boolean forms $x_1*(x'_1 \oplus x_2)$ x_1*x_2 and $x_1 \oplus (x_1*x_2)$ over the ordered pairs of the two-element Boolean algebra. 8

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