(b) Find the k-map for the following

(i) $(x' \wedge y' \wedge z) \vee (x' \wedge y \wedge z') \vee (x \wedge y \wedge z')$

(ii) $(x' \wedge y' \wedge z \wedge w) \vee (x' \wedge y \wedge z \wedge w')$
$\vee (x \wedge y' \wedge z \wedge w) \vee (x \wedge y \wedge z \wedge w')$. 8

10. (c) Use k-map to minimize the Boolean expression

(i) $f(a, b, c) = \Sigma\ 0, 2, 3, 7$

(ii) $f(a, b, c, d) = \Sigma 0, 1, 2, 3, 13, 15.$ 8

(d) For the Boolean expression represented by the following table, give the k-map representation also write the expression.

x	y	z	f(x, y, z)
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

8

First Semester M. Sc. (Part – I) (CBCS) Examination

(New Course)

MATHEMATICS

Paper – V

(Advanced Discrete Mathematics)

P. Pages : 6

Time : Three Hours] [Max. Marks : 80

Note : Solve one question from each unit.

UNIT I

1. (a) Prove that

$(\exists x)\ (P(x) \wedge Q(x)) \Rightarrow (\exists x)\ P(x) \wedge (\exists x)\ Q(x)$.
Is the converse true ? Justify. 8

(b) Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$, $\neg R \vee P$ and Q. Also state the rules of inferences. 8

2. (c) (i) Show that without truth table

$\sim(P \wedge Q) \rightarrow (\sim P \vee (\sim P \vee Q)) \Longleftrightarrow (\sim P \vee Q)$.

(ii) Express $P \rightarrow (\sim P \rightarrow Q)$ interms of $\uparrow$ only.
8

(d) State rules of specification and Generalization and show that $(\exists x)\ M(x)$ follows logically from the premises $(x)\ (H(x) \longrightarrow M(x))$ and $(\exists x)\ (H(x))$. 8

UNIT II

3. (a) Define :—

(i) Semi group

(ii) Monoids.

And show that the set N of natural number is semigroup under the operation $x * y = \max\{x, y\}$. Is it a monoid ? 8

(b) Define :

(i) Substitution property

(ii) Congruence relation on algebraic system.

Also, given the algebraic system $\langle N, + \rangle$ and $\langle z_4, +_4 \rangle$, where N is the set of natural no's and + is the operation of addition on N, show that there exists a homo-morphism from $\langle N, + \rangle$ to $\langle z_4, +_4 \rangle$. 8

4. (c) Define :

(i) Direct product

(ii) Free semigroup.

And let X be the set containing n elements let X^* denote the free semigroup generated by x and $\langle S, + \rangle$ be any other semigroup generated by any n-generators then show that there exists a homomorphism $g : X^* \longrightarrow S$. 8

(d) (i) Prove that for any commutative monoid $\langle M, * \rangle$ the set of idempotent elements of M forms a submonoid.

(ii) If $\langle S, * \rangle$ and $\langle T, \Delta \rangle$ are monoids with e_s and e_T as their identity elements respectively then their direct product $\langle SXT, 0 \rangle$ is also monoid with $\langle e_S, e_T \rangle$ as identity element. Prove this. 8

UNIT III

5. (a) Define :—

(i) Partial ordered relation

(ii) Lattice.

And show that, in a lattice if $a \leq b \leq c$ then $a + b = a * c$ and $(a * b) + (b * c) = b = (a + b) * (a + c)$. 8

(b) Give an example of a lattice which is neither distributive nor modular lattice and Justify. 8

6. (c) In a lattice prove that

(i) $(a * b) + (a * c) \leq a * [b + (a * c)]$

(ii) $(a + b) * (a + c) \geq a + [b * (a + c)]$ 8

(d) Let <L, ≤> be a lattice, for any a, b, c ∈ L then show that

(i) $a \vee (a \wedge b) = a$

(ii) $b \leq c \implies a \wedge b \leq a \wedge c$

$a \vee b \leq a \vee c$. 8

UNIT IV

7. (a) In a Boolean algebra <B, *, +, 1, 0, 1> show that

(i) $a = b \iff ab' + ba' = 0$

(ii) $a = 0 \iff ab' + a'b = b$

(iii) $a \leq b \implies a + bc = b(a + c)$ 8

(b) Let <p(s), ∩, ∪, ~, ϕ, s> be the algebra of the subsets of S = {a, b, c} and let g: p(s) ⟶B be a mapping onto the two element Boolean algebra given as B = {0, 1} such that g(x) = 1, if x contains the element b, otherwise g(x) = 0. Show that g is a Boolean homomorphism. 8

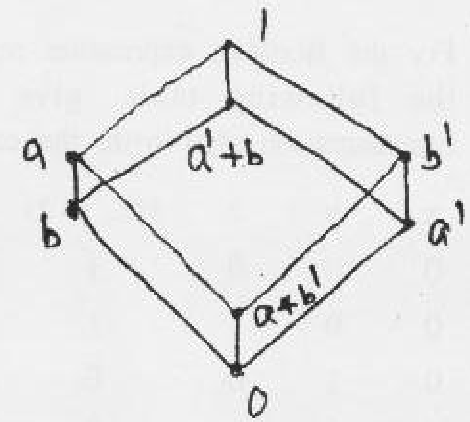
8. (c) Simplify the following Boolean expressions :

(i) $(a' * b' * c) \oplus (a * b' * c) \oplus (a * b' * c')$

(ii) $(a * c) \oplus c \oplus [(b \oplus b') * e]$ 8

(d) Consider the Boolean algebra given by the following diagram.

Let S_1 = {a, a,' 0, 1},
S_2 = {a∧b', b', a, 1},
S_3 = {a, b', 0, 1},
whether S_1, S_2, S_3 are sub-boolean algebra ?



8

UNIT V

9. (a) Write the following Boolean expression in an equivalent sum of product cannonical forms in three variables x_1, x_2, x_3 as

(i) $x_1 \oplus x_2$ (ii) $x_2 \oplus x_3'$. 8