M.Sc. (Part—I) Semester—I (C.B.C.S. Scheme) Examination
106 : MATHEMATICS
Paper—V
(Advanced Discrete Maths—I)

Time : Three Hours] [Maximum Marks : 80

Note :— Solve **ONE** question from each unit.

UNIT—I

1. (a) Translate into symbolic form the statement "jack and jill went up the hill" and construct the truth table and show that :

$$P \rightarrow (Q \rightarrow R) \Leftrightarrow (\urcorner Q \vee R) \Leftrightarrow (P \wedge Q) \rightarrow R.$$ 8

(b) State the Rules of Inferences (i.e. Rule P, Rule T and Rule CP) and show that :

$$(\urcorner P \wedge (\urcorner Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R.$$ 8

2. (c) Define and symbolise :

(i) Predicates

(ii) Universal specification

(iii) Quantifiers, also state Rules US, ES and UG

(iv) Existential Generalization. 8

(d) Prove that :

$$f(x)\ (P(x) \wedge Q(x)) \Rightarrow f(x)\ (P(x) \wedge f(x) \cdot Q(x)).$$

Is the converse true ? Explain. 8

UNIT—II

3. (a) Let A be the set of 2 × 2 matrices, show that semigroups $A = \left\{ \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \middle| a \in R \right\}$ and (R, +) are isomorphism. 8

(b) Show that, for any commutative monoid (M, ∗) the set of idempotent element of M forms a submonoid. 8

4. (c) Prove that <S, ∗> be a semigroup and R be a congruence relation on <S, ∗> then the quotient set S/R is a semigroup on <S/R, ⊕> where the operation ∗ on S. Also there exist a homomorphism from <S, ∗> onto <S/R, ⊕> and it is called natural homomorphism. 8

(d) Prove that <S, ∗> and <T, Δ> be two semigroups and g be a semigroup homomorphism from <S, ∗> to <T, Δ> corresponding to the homomorphism g, there exists a congruence relation R on <S, ∗> defined by XRY iff g(x) = g(y) for x, y ∈ S. 8

UNIT—III

5. (a) Prove that Pentagonal Lattice is not Modular Lattice. 8

(b) Define Distributive Lattice and show that the direct product of any two distributive lattice is distributive. 8

6. (c) Two lattices L & M be Modular if and only if $L \times M$ is Modular. 8

(d) Define chain and prove that every chain is distributive lattice. 8

UNIT—IV

7. (a) Define :

(i) Boolean homomorphism

(ii) Sub-Boolean Algebra and

Simplify the following Boolean Expressions :

(iii) $(a' * b' * c) \oplus (a * b' * c) \oplus (a * b' * c')$

(iv) $(a * c) \oplus c \oplus [(b \oplus b') * c]$ 8

(b) Let $<p(s), \cap, \cup, \sim, \phi, s>$ be the algebra of the subsets $s = \{a, b, c\}$ and let $g : p(s) \to B$ be a mapping onto the two element Boolean algebra given as $B = \{0, 1\}$ such that $g(x) = 1$, if x contains the element otherwise $g(x) = 0$ then show that g is a Boolean Homomorphism. 8

8. (c) Prove that the intersection of any two subalgebra of a Boolean algebra B is also subalgebra of B. 8

(d) Prove that the following Boolean identifies :

(i) $a \oplus (a' * b) = (a \oplus b)$

(ii) $a * (a' \oplus b) = a * b.$ 8

UNIT—V

9. (a) Write the following Boolean expressions in an equivalent sum of products canonical form in three variables x_1, x_2 and x_3 :

(i) $x_1 * x_2$

(ii) $x_1 \oplus x_2$. 8

(b) Obtain the product of sums canonical forms of the expression $x_1 x'_2 + x_3$. 8

10. (c) Simplify the following Boolean expressions using K-map :

(i) $E(x, y, z) = x'y'z' + xy'z'$

(ii) $E(x, y, z) = xyz' + xyz$. 8

(d) Obtain the product of sums canonical forms of the expression :

$[(x_1 + x_2)(x_3 \cdot x_4)']'$. 8