First Semester M. Sc. (Part–I) (CBCS) Examination

(Old Course)

MATHEMATICS

Paper : (105)

Differential Geometry (Optional)

P. Pages : 5

Time : Three Hours] [Max. Marks : 80

Note : Solve **One** question from each unit.

UNIT I

1. (a) Show that metric is invariant under parametric transformation $u' = \phi(u, v)$, $v' = \psi(u, v)$ but the coefficients E, F and G are not invariants. 8

   (b) If (l, m) and (l', m') are the direction coefficients of two directions at a point P on the surface and θ is the angle between the two directions at P, then prove that :

   (i) $\cos\theta = Ell' + F(lm' + l'm) + Gmm'$.

   (ii) $\sin\theta = H(lm' - l'm)$. 8

(c) Prove that a proper parametric transformation either leaves every normal unchanged or reverses the direction of the normal. 8

(d) A surface of revolution is defined by the equations

$x = \cos u \cos v, \; y = \cos u \sin v,$

$z = -\sin u + \log [\tan (\pi/4 + v/2)],$

Where $0 < u < \pi/2$ and $o < \upsilon < 2\pi$, then prove that, the metric is $ds^2 = \tan^2 u \; du^2 + \cos^2 u \; d\upsilon^2$. 8

UNIT II

3. (a) State and prove second existence theorem for a region of a surface of class 'r'. 8

(b) Prove that the curves of the family υ^3/u^2 = constant, are geodesic, on a surface with the metric $\upsilon^2 \; du^2 - 2u\upsilon du d\upsilon + 2u^2 d\upsilon^2$; $u > 0, \; \upsilon > 0$. 8

4. (c) Prove that : on a general surface, a necessary and sufficient condition that the curve $u = c$ is a geodesic if $G \, G_1 + FG_2 - 2\,GF_2 = 0$. 8

(d) Prove that necessary and sufficient condition for a curve to be a geodesic is

$$U \frac{\partial t}{\partial \upsilon} - V \frac{\partial t}{\partial u} = 0$$

8

UNIT III

5. (a) State and prove Gauss–Bonnet theorem. 8

(b) If P is given point on a surface and triangle is the area of the geodesic triangle ABC containing P, then prove that, the Gaussian curvature K at p is

$$K = \frac{A + B + C - \pi}{\Delta}$$

When the limit is taken as the vertices A, B and C tends to P. 8

6. (c) Define :—

(i) Geodesic

(ii) Geodesic curvature.

(iii) Surface of constant curvature.

(iv) Conformal mapping. 4

(d) State and prove the Liouville's formula for geodesic curvature. 6

(e) Prove that the total curvature of the whole surface of an anchor–ring is zero. 6

UNIT IV

7. (a) In order that η^{r+s} number

$$\urcorner^{i_1, i_2 \cdots\cdots i_r}_{j_1 \cdot j_2 \cdots\cdots j_s}$$

associated with each basis of V^1 can be regarded as the components of tensor T of type (r, s), the necessary and sufficient condition is that for any 'r' covariant vector's λ, μ, ----v, the expression

$$T^{i_1 \cdot i_2 \cdots\cdots ir}_{j_1 \cdot j_2 \cdots\cdots j_s} \cdot \lambda^{j_1} \cdot \mu^{j_2} \cdots\cdots v^{j_s} \cdot \alpha_{i_1}, \beta_{i_2} \cdots Y_{i_r}$$

Shall be invariant under a change of basis of v. Prove this. 10

(b) If $P_{\lambda\mu v}$ is such that $A^{\lambda} P_{\lambda\mu v}$ is tensor for any vector A^{λ}. Prove that $P_{\lambda\mu v}$ is also a tensor. 6

8. (c) Prove that, if a_{ij} is a symmetric covariant tensor and b_k is the covariant tensor, which satisfy $a_{ij}\, b_k + a_{jk}\, b_i + a_{ki}\, b_j = 0$, then $a_{ij} = 0$ or $b_k = 0$. 6

(d) Show that the basis (e_i) of a vector space V induces a unique basis for its dual space. 10

UNIT V

9. (a) Show that the connexion coefficients are not the components of a tensor. 8

(b) Prove that :—

(i) $(A^{ij} + B^{ij})_{,k} = A^{ij}_{,k} + B^{ij}_{,k}$

(ii) $(A^i_j B_k)_{,l} = A^i_{j,l} B_k + A^i_j B_{k,l}$ 8

10. (c) Prove that the connexion coefficient are not the components of a tensor. 8

(d) Prove that the successive covariant differentiation of a scalar are commutative only when connexion has zero torsion. 8