AU–220

M.Sc. (Part—I) Semester—I (C.B.C.S. Scheme) Examination

MATHEMATICS

Paper—105

(Differential Geometry)

Time : Three Hours] [Maximum Marks : 80

N.B. :— Solve **ONE** question from each unit.

UNIT—I

1. (a) Prove that the position vector of a point on the anchor ring is

$$r = ((b + a \cos u) \cos v, (b + a \cos u) \sin v, a \sin u))$$

where (b, 0, 0) is the centre of the circle and z-axis is the axis of rotation. 6

(b) Prove that the first fundamental form of a surface is a positive definite quadratic form in du, dv. 6

(c) Find E, F, G and H for the paraboloid $x = u$, $y = v$, $z = u^2 - v^2$. 4

2. (p) If (l, m) and (l', m') are the direction coefficients of two directions at a point P on the surface and θ is the angle between the two direction at P, then prove that,

(i) $\cos \theta = Ell' + F(lm' + l'm) + Gmm'$

(ii) $\sin \theta = H(lm' - l'm)$. 6

(q) Obtain the surface equation of a cone with semi-vertical angle α and find the singularities, parametric curves, tangent plane at a point and the surface normal. 6

(r) If w is the angle between the parametric curves at the point of intersection, then prove that

$$\tan w = \frac{H}{F}.$$ 4

UNIT—II

3. (a) Prove that the necessary and sufficient condition for a curve to be a geodesic is
$$U\frac{\partial T}{\partial \dot{v}} - V\frac{\partial T}{\partial \dot{u}} = 0$$ 8

(b) Find surface of revolution which is isometric with a region of the right helicoid. 8

4. (p) Prove that on the general surface, a necessary and sufficient condition that the curve $u = c$ be a geodesic is
$$GG_1 + FG_2 - 2GF_2 = 0.$$ 8

(q) Show that the curves bisecting the angles between the parametric curves are given by $Edu^2 - Gdv^2 = 0$, using that direction coefficients of the parametric curves v = constant and u = constant are $\left(\frac{1}{\sqrt{E}}, 0\right)$ and $\left(0, \frac{1}{\sqrt{G}}\right)$ respectively. 8

UNIT—III

5. (a) If U and V are the intrinsic quantities of a surface at a point (u, v) then prove that

(i) $K_g = \frac{1}{H}\frac{V(s)}{u'}$

(ii) $K_g = -\frac{1}{H}\frac{V(s)}{v'}$ 8

(b) State and prove Gauss-Bonnet theorem. 8

6. (p) Obtain intrinsic formula for Gaussian curvature K for a surface having orthogonal parametric curves and hence find Gaussian curvature at any point of a sphere $\bar{r} = a(\sin u \cos v, \sin u \sin v, \cos u)$ where $0 < u < \pi$, $0 < v < 2\pi$. 8

(q) Prove that for any curve on a surface, the geodesic curvature vector is intrinsic. 8

UNIT—IV

7. (a) If V is n-dimensional vector space and W be the m-dimensional vector space then show that dimension of tensor product $V \otimes W$ is mn. 8

(b) Prove that

(i) $T^{i'\alpha'} = P^{i'}_{i} P^{\alpha'}_{\alpha} T^{i\alpha}$

(ii) $T^{i\alpha} = P^{i}_{i'} P^{\alpha}_{\alpha'} T^{i'\alpha'}$ 8

8. (p) Prove that there is a natural isomorphism relating V and V**, the dual space of V*. 8

(q) Show that the basis (e_i) of vector space V induces a unique basis in V*. 8

UNIT—V

9. (a) Prove that the successive covariant differentiation of a scalar are commutative only when connexion has zero torsion. 8

(b) Show that $\left(A^{ij} + B^{ij}\right)_{,k} = A^{ij}_{,k} + B^{ij}_{,k}$, where (,) denotes covariant differentiation. 8

10. (p) Show that the connexion coefficients are not components of a tensor. 8

(q) Prove that $\left(A^{i}_{j}\ B_{k}\right)_{,l} = A^{i}_{j,l}\, B_{k} + A^{i}_{j}\, B_{k,l}$, where (,) denotes covariant differentiation. 8