AV–1873

**M.Sc. (Part—I) Semester—I (C.B.C.S. Scheme) Examination**

**105 : MATHEMATICS**

**(Differential Geometry)**

Time : Three Hours] [Maximum Marks : 80

**N.B. :—** Solve one question from each unit.

## UNIT—I

1. (a) Find the surface area of anchor ring given by

$$\bar{r} = ((b + a\cos u)\cos v, (b + a\cos u)\sin v, a\sin u),$$

where $0 \le u \le 2\pi$ and $0 \le v \le 2\pi$. 6

(b) Explain :

(i) Representation of a right helicoid.

(ii) Representation of the general helicoid. 6

(c) Find the parametric directions and the angle between the parametric curves. 4

2. (p) Prove that the position vector of any point on the surface of revolution generated by the curve $[g(u), 0, f(u)]$ in the XOZ plane is $r = (g(u)\cos v, g(u)\sin v, f(u))$, where v is angle of rotation about z-axis. 6

(q) Prove that the metric $E\,du^2 + 2F\,du dv + G\,dv^2$ is invariant under a parametric transformation. 6

(r) For the cone with vertex at the origin and semi-vertical angle $\alpha$, prove that the tangent plane is the same at all points on the generating line. 4

## UNIT—II

3. (a) A helicoid is generated by the screw motion of a straight line skew to the axis. Find the curve coplanar with the axis which generates the same helicoid. 8

(b) Prove that the curves of the family $\frac{v^3}{u^2}$ = constant are geodesics on a surface with the metric $v^2du^2 - 2uv\,dudv + 2u^2dv^2$, $u > 0$, $v > 0$. 8

4. (p) Prove that any curve $u = u(t)$, $v = v(t)$ on a surface $r = r(u, v)$ is a geodesic if and only if the principal normal at every point on the curve is normal to the surface. 8

(q) Show that on a right helicoid, the family of curves orthogonal to $u\cos v$ = constant is the family $(u^2 + a^2)\sin^2 v$ = constant. 8

## UNIT—III

5. (a) Prove that, the components $\lambda$, $\mu$ of the geodesic curvature vector are given by :

$$\lambda = \frac{1}{H^2}\frac{U}{v'}\frac{\partial T}{\partial v'} = -\frac{1}{H^2}\frac{V}{u'}\frac{\partial T}{\partial v'}$$

$$\mu = \frac{1}{H^2}\frac{V}{u'}\frac{\partial T}{\partial u'} = -\frac{1}{H^2}\frac{U}{v'}\frac{\partial T}{\partial u'}$$

with s as a parameter. 8

(b) Find the Gaussian curvature at a point (u, v) of the anchor ring

$$r = ((b + a \cos u) \cos v, (b + a \cos u) \sin v, a \sin u)$$

where $0 < u, v < 2\pi$. 8

6\. (p) Prove that two surfaces of the same constant curvature are locally isometric. 8

(q) Prove that, if a mapping of a surface S onto a surface S* is both geodesie and conformal, then it is an isometry or a similarity mapping. 8

## UNIT—IV

7\. (a) Show that in vector space the old components can be expressed in terms of new components and new components can be expressed in terms of old components. 8

(b) Explain :

(i) Dual Space

(ii) Contraction of Tensor. 8

8\. (p) Prove that, in order that $n^{r+s}$ number $T^{i_1, i_2, \ldots, i_r}_{j_1, j_2, \ldots, j_s}$ associated with each basis of v′ can be regarded as the components of tensor T of type (r, s), it is necessary and sufficient that for any r covariant vectors α, β, .... γ the expression :

$$T^{i_1, i_2, \ldots, i_r}_{j_1, j_2, \ldots, j_s} \lambda^{j_1} \mu^{j_2} \ldots \nu^{j_s} \cdot \alpha_{i_1} \beta_{i_2} \ldots \gamma_{i_r}$$

shall be invariant under a change of basis of v′. 8

(q) Prove that any tensor of second order can be expressed as sum of symmetric tensor and skew-symmetric tensor. 8

## UNIT—V

9\. (a) Show that two operations of contraction and covariant differentiation are commutative. 8

(b) Show that :

$$(A_{ij} - B_{ij})_{,k} = A_{ij,k} - B_{ij,k},$$

where (,) denotes covariant differentiation. 8

10\. (p) Prove that covariant differentiation maps tensor field of class r and type (m, p) into the tensor field of (r − 1) and type (m, p + 1). 8

(q) Prove that the differentiation property implies that the space of tangent vectors is n-dimensional. 8