

UNIT IV

7. (a) (i) Define residue of a function. Hence find the value of $\int_r e^{2/z} dz$ where r is $|z|=1$.
 (ii) Suppose f has a pole of order m at $z=a$ and put $g(z)=(z-a)^m f(z)$; then prove that $\text{Res}(f; a) = \frac{1}{(m-1)!} g^{(m-1)}(a)$. 4+4

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{3+2\cos\theta}$. 8

8. (c) State and prove Hadamard's three circle theorem. 8
 (d) Show that $\int_0^\infty \frac{x^{-c}}{1+x} dx = \frac{\pi}{\sin \pi c}$ if $0 < c < 1$. 8

UNIT V

9. (a) If $\{f_n\}$ is a sequence in $H(G)$ and f belongs to $C(G; \mathbb{C})$ such that $f_n \rightarrow f$ then prove that f is analytic and also prove that $f_n^{(k)} \rightarrow f^{(k)}$ for each integer $k \geq 1$. 8

First Semester M.A./M.Sc. (Part - I) (CBCS)
Examination

(Old)

MATHEMATICS

103 - Complex Analysis - I

P. Pages : 5

Time : Three Hours]

[Max. Marks : 80

Note : Solve One question from each unit.

UNIT I

1. (a) Let $f: G \rightarrow \mathbb{C}$ be analytic and suppose $\bar{B}(a; r) \subset G$ ($r > 0$). If $r(t) = a + re^{it}$, $0 \leq t \leq 2\pi$; then prove that

$$f(z) = \frac{1}{2\pi i} \int_r \frac{f(w)}{w-z} dw, \text{ for } |z-a| < r$$

Also evaluate $\int_r \frac{z^2}{4-z^2} dz$ where r is $|z+1|=2$

6+2

- (b) (i) If f is analytic in $B(a, R)$ and suppose that r is a closed curve in $B(a, R)$. Then prove that $\int_r f(z) dz = 0$. 4

(ii) Evaluate $\int_{|z|=4} \frac{e^z \cos z}{(z-\pi)^3} dz.$ 4

2. (c) State and prove Liouville's theorem. Also show that $f(z) = \sin z$, $z \in \mathbb{C}$ is not bounded. 6+2

(d) (i) If $g(w) = \int_c \frac{2z^2 - z - 2}{z^w} dz$ then show that

$g(2) = 8\pi i$. What is the value of $g(w)$ when $|w| > 3$. 4

- (ii) Also prove that analytic functions are infinitely differentiable. 4

UNIT II

3. (a) State and prove Morera's theorem. 8

(b) (i) Expand $f(z) = \frac{1}{z^2 + 4z + 3}$ in $|z| < 1$ and

(ii) Expand $f(z) = \sin^3 z$ about $z=0$. 4+4

4. (c) State and prove Schwarz's lemma. 8

- (d) Let G be a region and let f be an analytic function on G with zeros at $a_1, a_2, \dots, a_m$. (repeated according to multiplicity). If γ is a closed rectifiable curve in G which does not pass through any point a_k then prove that

$$\frac{1}{2\pi i} \int_{\gamma} \frac{f'(z)}{f(z)} dz = \sum_{k=1}^m n(\gamma; a_k).$$
 8

UNIT III

5. (a) If f has an isolated singularity at 'a' then prove that the point $z=a$ is a removable singularity iff $\lim_{z \rightarrow a} (z-a) f(z) = 0$. 8

- (b) Show that the function $f(z) = z^4 - 7z - 1$ has three zeros inside the ann $(0; 1, 2)$ 8

6. (c) State and prove Casorti-wierstrass theorem. 8

(d) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in the Laurent's series

for the following region.

(i) $|z| < 1$

(ii) $1 < |z| < 3$ and

(iii) $|z| > 3$. 8

- (b) Let f be an entire function and let $\{a_n\}$ be the non-zero zeros of f repeated according to multiplicity; suppose f has a zero at $z=0$ of order $m \geq 0$ then prove that there is an entire function 'g' and a sequence of integers $\{P_n\}$ such that

$$f(z) = z^m \cdot e^{g(z)} \prod_{n=1}^{\infty} E_{P_n}(z/a^n). \quad 8$$

10. (c) If $|z| \leq 1$ and $p \geq 0$ then prove that

$$|1 - E_p(z)| \leq |z|^{p+1} \quad 8$$

- (d) Show that :—

- (i) $H(G)$ is a complete metric space and
 (ii) If $\{f_n\} \subset H(G)$ converges to F in $H(G)$ and each f_n never vanishes on G then either $f \equiv 0$ or f never vanishes on G . 4+4



