UNIT IV

7. (a) (i) Define residue of a function. Hence find the value of $\int_{r} e^{2/z}\,dz$ where r is $|z|=1$.

(ii) Suppose f has a pole of order m at z=a and put $g(z)=(z-a)^{m}f(z)$; then prove that $\text{Res }(f;\,a) = \frac{1}{(m-1)!}\,g^{(m-1)}(a)$. 4+4

(b) Evaluate $\int_{0}^{2\pi} \frac{d\theta}{3+2\cos\theta}$. 8

8. (c) State and prove 'Hadamard's three circle theorem. 8

(d) Show that $\int_{0}^{\infty} \frac{x^{-c}}{1+x}\,dx = \frac{\pi}{\sin \pi c}$ if $0<c<1$. 8

UNIT V

9. (a) If $\{f_n\}$ is a sequence in H(G) and f belongs to $C(G;\mathbb{C})$ such that $f_n \to f$ then prove that f is anlaytic and also prove that $f_n^{(k)} \to f^{(k)}$ for each integer $k \geq 1$. 8

First Semester M.A./M.Sc. (Part – I) (CBCS) Examination

(Old)

MATHEMATICS

103 – Complex Analysis – I

P. Pages : 5

Time : Three Hours] [Max. Marks : 80

Note : Solve **One** question from each unit.

UNIT I

1. (a) Het $f : G \to \mathbb{C}$ be analytic and suppose $\bar{B}(a;\,r) \subset G$ $(r > 0)$. If $r(t) = a + re^{it}$; $0 \leq + \leq 2\pi$; then prove that

$$f(z) = \frac{1}{2\pi i} \int_{r} \frac{f(w)}{w-z}\,dw, \quad \text{for } |z-a|<r$$

Also evaluate $\int_{r} \frac{z^2}{4-z^2}\,dz$ where r is $|z+1|=2$ 6+2

(b) (i) If f is analytic in B(a, R) and suppose that r is a closed curve in B(a, R). Then prove that $\int_{r} f(z)\,dz = 0$. 4

(ii) Evaluate $\int_{|z|=4} \frac{e^z \cos z}{(z-\pi)^3}\, dz.$ 4

2. (c) State and prove Liouville's theorem. Also show that $f(z) = \sin z,\ z \in \mathbb{C}$ is not bounded. 6+2

(d) (i) If $g(w) = \int_c \frac{2z^2 - z - 2}{z^{-w}}\, dz$ then show that $g(2) = 8\pi i$. What is the value of $g(w)$ when $|w| > 3$. 4

(ii) Also prove that analytic functions are infinitely differentiable. 4

UNIT II

3. (a) State and prove Morera's theorem. 8

(b) (i) Expand $f(z) = \frac{1}{z^2 + 4z + 3}$ in $|z| < 1$ and

(ii) Expand $f(z) = \sin^3 z$ about $z = 0$. 4+4

4. (c) State and prove Schwarz's lemma. 8

(d) Let G be a region and let f be an analytic function on G with zeros at a_1, a_2 ----- a_m. (repeated according to multiplicity). If r is a closed rectifiable curve in G which does not pass through any point a_k then prove that

$$\frac{1}{2\pi i} \int_r \frac{f^1(z)}{f(z)}\, dz = \sum_{k=1}^{m} n(r; a_k).$$ 8

UNIT III

5. (a) If f has an isolated singularity at 'a' then prove that the point $z = a$ is a removable singularity iff $\lim_{z \to a} (z-a)\, f(z) = 0$. 8

(b) Show that the function $f(z) = z^4 - 7z - 1$ has three zeros inside the ann (0; 1, 2) 8

6. (c) State and prove Casorti-wierstrass theorem. 8

(d) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in the Laurent's series for the following region.

(i) $|z| < 1$

(ii) $1 < |z| < 3$ and

(iii) $|z| > 3$. 8

(b) Let f be an entire function and let $\{a_n\}$ be the non-zero zeros of f repeated according to multiplicity; suppose f has a zero at $z=0$ of order $m \geq 0$ then prove that there is an entire function 'g' and a sequence of integers $\{P_n\}$ such that

$$f(z) = z^m . e^{g(z)} \prod_{n=1}^{\infty} E_{P_n} (z/a^n). \qquad 8$$

10. (c) If $|z| \leq 1$ and $p \geq 0$ then prove that

$$|1 - E_P(z)| \leq |z|^{P+1} \qquad 8$$

(d) Show that :—

(i) H(G) is a complete metric space and

(ii) If $\{f_n\} C H (G)$ converges to F in H (G) and each f_n never vanishes on G then either $f \equiv 0$ or f never vanishes on G. 4+4

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