Show that the minimum value of $f(x, y, z)$ is given by

$$x^2 = \frac{u}{2a(u+a)};\quad y^2 = \frac{u}{2b(u+b)};\quad z^2 = \frac{u}{2c(u+c)}$$

where u is the positive root of the equation. $u^3 - (bc+ca+ab)u - 2abc = 0$ (schlomilch). 8

AQ – 805

First Semester M.Sc. (Part – I) (CBCS) Examination

(New Course)

MATHEMATICS

Paper – I

Real Analysis

P. Pages : 6

Time : Three Hours] [Max. Marks : 80

Note : Solve any **One** question from each unit.

UNIT I

1. (a) (i) If f is continuous on [a, b] then prove that $f \in R(\alpha)$ on [a, b]. 4

(ii) If P^* is a refinement of P them prove that $U(P^*, F, \alpha) \le U(P, f, \alpha)$ 4

(b) Define two functions β_1, β_2 as follows $\beta_j(x) = 0$ if $x < 0$, $\beta_j(x) = 1$ if $x > 0$ for $j = 1, 2$, and $\beta_1(0) = 0$, $\beta_2(0) = 1$.

Let F be a bounded function on [–1, 1]

(i) Prove that $f \in R(\beta_1)$ if and only if $f(0^+) = f(0)$ and then $\int f d\beta_1 = f(0)$

(ii) State a similar result for β_2. 8

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2\. (c) Define a rectifiable curve. Given a curve γ whose derivative γ' is continuous on [a, b]. Show that γ is a rectifiable curve and has length $\int_a^b |\gamma'(t)|\,dt$. 10

(d) State and prove fundamental theorem of integral calculus for vector valued function. 6

UNIT II

3\. (a) State and prove Cauchy's criterion for uniform convergence of a sequence of a functions $\{fn(x)\}$. 6

(b) Given $\{fn(x)\}$ is a sequence of continuous functions defined on a compact set E converging to a continuous function f(x) on E. Assume $fn(x) \geq f_{n+1}(x)$, $\forall x \in E$ $n = 1, 2, 3, \ldots$. Then show that $fn(x) \to f(x)$ uniformly on E. 10

4\. (c) Prove that there exists a real continuous function on the real line which is nowhere differentiable. 10

(d) If X is a metric space, C(x) will denote the set of all complex valued continuous functions with domain X. Prove that C(x), the metric space is complete. 6

UNIT III

5\. (a) Suppose the series $\sum_{n=0}^{\infty} C_n x^n$ converges for $|x| < R$ and define.

$$f(x) = \sum_{n=0}^{\infty} C_n x^n, \qquad (|x| < R)$$

Then prove that $\sum_{n=0}^{\infty} c_n x^n$ converges uniformly on $[-R+\epsilon, R-\epsilon]$, no matter which $\epsilon > 0$ is chosen. Prove that f is continuous and differentiable in $(-R, R)$ and

$$f'(x) = \sum_{n=1}^{\infty} nc_n x^{n-1} \qquad (|x| < R).$$ 8

(b) Prove that the product of two convergent series converges if atleast one of the series converges absolutely. 8

6\. (c) State and prove Abel's theorem for the power series. 8

(d) Prove or disprove that "The product of two convergent series is always convergent". 8

UNIT IV

7. (a) For $A \in L(R^n, R^m)$, set of all linear transformations of vector space R^n into vector space R^m, define the norm $\|A\|$.
Hence prove

(i) If $A \in L(R^n, R^m)$ then $\|A\| < \infty$ and A is uniformly continuous mapping of R^n into R^m.

(ii) If A, $B \in L(R^n, R^m)$ and C is a scalar then prove that

$\|A+B\| \leq \|A\| + \|B\|$ and

$\|CA\| \leq |C| . \|A\|$ 8

(b) Suppose E be an open set in R^n and $f : E \to R^m$ and $x \in E$.
Define differentiability of f at $x \in E$. If f is differentiable at $x \in E$ then prove that it is unique. 8

UNIT IV

8. (c) Suppose f is defined in an open set $E C R^2$. Suppose $D_1 f$; $D_{21} f$ and $D_2 f$ exist, at every point of E and D_{21} f is continuous at some point $(a, b) \in E$. Then prove that $D_{12} f$ exists at (a, b) and $(D_{12} f)(a, b) = (D_{21} f)(a, b)$. 8

(d) Suppose f maps a convex open set $E \subset R^n$ into R^m, f is differentiable in E and there is a real number M such that $\|f'(x)\| \leq M$ for every $x \in E$. Then prove that $|f(b)-f(a)| \leq M |b-a|$. If, in addition $f'(x)=0$ for all $x \in E$ then prove that f is constant. 8

UNIT V

9. (a) State the prove Implicit function theorem. 10

(b) If f maps an open set $E \subset R^n$ into R^m. Define Jacobian of f at x. If $U=\cos x$, $V=\sin x. \cos y$ and $w=\sin x. \sin y. \cos z$ then show that

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = (-1)^2 \sin^3 x . \sin^2 y. \sin z$$ 6

10. (c) Let A be an open subset of R^n and assume that $f: A \to R^n$ has continuous partial derivatives $D_j f_i$ on A. If Jacobian determinant If $(x) \neq 0$ for all x in A, then prove that f is an open mapping. 8

(d) If $f(x,y,z) = (a^2x^2+b^2y^2+c^2z^2)/x^2y^2z^2$, where $ax^2+by^2+cz^2=1$ and a, b, c are positive.

Show that the minimum value of $f(x, y, z)$ is given by

$$x^2 = \frac{u}{2a(u+a)}; \quad y^2 = \frac{u}{2b(u+b)}; \quad z^2 = \frac{u}{2c(u+c)}$$

where u is the positive root of the equation. $u^3-(bc+ca+ab)u-2abc=0$ (schlomilch). 8