UNIT IV

7. (a) Prove that every compact subset of a Hausdroff space is closed. 8

(b) Show that in a T_2-space, a convergent sequence has a unique limit. 8

8. (c) Show that the property of being a Ist axiom space is hereditary property. 8

(d) Prove that a topological space X is a To-space if the closures of distinct points are distinct. 8

UNIT V

9. (a) Show that every regular Lindelöf space is normal. 6

(b) State and prove Urysohn's Lemma. 10

10. (c) Show that regularity is hereditary property. 8

(d) Prove that Every compact Hausdroff space is normal. 8

AQ – 801

First Semester M. Sc. (Part – I) (C. B. C. S - Pattern) Examination

(Old Course)

MATHEMATICS

Paper – IV

Topology – I

P. Pages : 4

Time : Three Hours] [Max. Marks : 80

Note : Solve **one** question from each unit.

UNIT I

1. (a) Prove that If $A \lesssim B$ and $B \lesssim A$ then $A \sim B$. 8

(b) Show that $\aleph_0 \aleph_0 = \aleph_0$, $\aleph_0 C = C$, $CC = C$ where, $\aleph_0$ is cardinality of countable set. 8

2. (c) Show that the set of all real numbers is uncountable. 8

(d) Show that each ordinal number "α" is the order type of the set $W\alpha$, where $W\alpha$ is the set of all ordinal numbers. 8

UNIT II

3. (a) Prove the followings :—

(i) $e(\phi) = X$

(ii) $e(E) \subseteq CE$

(iii) $e(E) = e(ce(E))$, and

(iv) $e(A \cup B) = e(A) \cap e(B)$ 8

(b) Prove that the family f of all closed subsets in a topological space has the following properties.

(i) The intersection of any number of members of f is a member of $\overline{f}$.

(ii) The union of any finite number of members of f is a member of $\overline{f}$. 8

4. (c) Define topological space and

Let $x = \{1, 2, 3, 4, 5\}$ and

$J = \{\phi, \{2\}, \{3, 4\}, \{2, 3, 4\}, \{1, 3, 4\} \{1, 2, 3, 4\}, X\}$ then

find exterior of

(i) $A = \{3\}$

(ii) $B = \{1, 2\}$. 8

(d) If A and B are subset of topological space (X, J). Then prove that

$d(A \cup B) = d(A) \cup d(B)$

$d(A \cap B) \subseteq d(A) \cap d(B)$ 8

UNIT III

5. (a) Define compact set and show that every closed subset of a compact space is compact. 8

(b) Prove that a compact subset of a topological space is countably compact. 8

6. (c) Define the following term :—

(i) Open mapping

(ii) Closed mapping

(iii) Homeomorphism

(iv) Continuous mapping. 8

(d) Show that the components of a topological space (X, J) are closed subset of X. 8