converges to a point $x \in X$, then show that x is a limit point of the set E. 8

UNIT V

9. (a) Show that a topological space X is normal iff for any closed set F and open set G containing F, there exist an open set G* such that $F \subseteq G^*$ and $C(G^*) \subseteq G$. 8

   (b) Show that regularity is a topological property. 8

10. (c) Show that every regular T_0-space is a T_3-space. 8

   (d) Explain the following terms :—

   (i) Regular space

   (ii) Normal space

   (iii) Completely regular space. 8

First Semester M. Sc. (Part – I) (CBCS - Pattern) Examination

(New Course)

MATHEMATICS

Paper – IV

Topology - I

P. Pages : 4

Time : Three Hours] [Max. Marks : 80

Note : Solve one question from each unit.

UNIT I

1. (a) State and prove Cantor theorem. 8

   (b) Show that addition of order type is not commutative. 8

2. (c) Show that every infinite set is equipotent to a proper subset of itself. 8

   (d) Define denumerable set and show that $2^{\aleph_0} = C$. 8

UNIT II

3. (a) If A, B and E are subsets of the topological space (X, J), then show that the derived set has the following properties

(i) If $A \subseteq B$, then $d(A) \subseteq d(B)$

(ii) If $x \in d(E)$ then $x \in d(E) \setminus \{x\})$ 8

(b) Define base and subbase for a topology. Prove that if E is a subset of subspace (x^*, J^*) of a topological space (X, J) then $C^*(E) = X^* \cap C(E)$. 8

4. (c) Show that for any set E in a topological space $C(E) = E \cup d(E)$, where 'C' is a closure operator. 8

(d) State the following :—

(i) Exterior axioms

(ii) Interior axioms

(iii) Kuratowski closure axioms. 8

UNIT III

5. (a) If E is a subset of a subspace (X^*, J^*) of a topological space (X, J) then show that E is a J^*-compact iff J-is compact. 8

(b) Show that if every two points of a set E are contained in some connected subset of E, then E is a connected set. 8

6. (c) Prove that a mapping f of X intoX^* is open if and only if $f(i(E)) \subseteq i^*(f(E))$. for every $E \subseteq X$. 8

(d) If $E = A|B$ is closed then show that A and B are closed. 8

UNIT IV

7. (a) Define convergent sequence and show that in a Hausdroff space, a convegent sequence has a unique limit. 8

(b) Prove that in a T_1 space X, a pt x is a limit of a set E iff every open set containing x contains an infinite number of distinct points of E. 8

8. (c) Show that every subspace of T_1-space is T_1-space and T_0-space is T_0-space. 8

(d) If $<x_n>$ is a sequence of distinct points of a subset E of a topological space X which

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