AU–219

M.Sc. (Part—I) Semester—I (C.B.C.S. Scheme) Examination

104 : MATHEMATICS

(Topology—I)

Time : Three Hours] [Maximum Marks : 80

N.B. :— Solve **ONE** question from each Unit.

UNIT—I

1. (a) Define :

   (i) Similar sets

   (ii) Denumerable set

   (iii) Well-ordered set. 6

   (b) State and prove Schoeder-Bernstein Theorem. 10

2. (c) If α and β are ordinal numbers then show that either $\alpha \leq \beta$ or $\beta \leq \alpha$. 8

   (d) Prove that $2^a > a$ for every cardinal number a. 8

UNIT—II

3. (a) Show that the union of two topologies for a set need not be a topology for the set, but the intersection of any family of topologies for a set will be a topology for that set. 2+6

   (b) Prove that the four Kuratowski closure axioms may be replaced by the single condition $A \cup c(A) \cup c(c(B)) = c(A \cup B) - c(\phi)$ for all subsets A, B $\subseteq$ X. Here C is the closure operation on a topological space X. 8

4. (c) Show that for any set E in a Topological space $(X, \mathcal{T})$, $i(E) = C \subset (CE)$. 8

   (d) Prove that a family **B** of sets is a base for a topology for the set $X = \cup \{B : B \in \mathbf{B}\}$ iff for every $B_1, B_2 \in \mathbf{B}$ and every $x \in B_1 \cap B_2$, there exists a $B \in \mathbf{B}$ such that $x \in B \subseteq B_1 \cap B_2$. 8

UNIT—III

5. (a) Define separation of a set. If C is a connected subset of a topological space $(X, \mathcal{T})$ which has a separation $X = A/B$ then show that either $C \subseteq A$ or $C \subseteq B$. 2+6

(b) Prove that :

(i) Continuous image of a connected set is connected, and

(ii) Continuous image of a compact set is compact. 4+4

6. (c) Show that a topological space $(X, \mathcal{T})$ is compact iff any family of closed sets having the finite intersection property has a nonempty intersection. 10

(d) Prove that a mapping f of X into X^* is open iff $f(i(E)) \subseteq i^*(f(E))$ for every $E \subseteq X$. 6

UNIT—IV

7. (a) Show that a topological space X is a T_0-space iff the closures of distinct points are distinct. 8

(b) Show that every compact subset E of a Hausdorff space X is closed. 8

8. (c) Prove that in a T_1-space X, a point x is a limit point of a set E iff every open set containing x contains an infinite number of distinct points of E. 8

(d) If x is a point and E a subset of a T_1-space X satisfying the first axion of countability then show that x is a limit point of E iff there exists a sequence of distinct points in E converging to x. 8

UNIT—V

9. (a) Prove that a topological space X is regular iff for every point $x \in X$ and open set G containing x there exists an open set G^* such that $x \in G^*$ and $c(G^*) \subseteq G$. 8

(b) Show that a normal space is completely regular iff it is regular. 8

10. (c) Show that a topological space X is completely normal iff every subspace of X is normal. 10

(d) If x and y are two distinct points in a Tichonov space X, then prove that there exists a real-valued continuous mapping f of X such that $f(x) \neq f(y)$. 6