**AV–1872**

**M.Sc. Part–I Semester–I (CBCS Scheme) Examination**

**MATHEMATICS**

**(Topology–I)**

**Paper–104**

Time : Three Hours] [Maximum Marks : 80

**N.B.:**—Solve **ONE** question from each unit.

**UNIT—I**

1. (a) Prove the following :

(i) The union of a denumerable number of denumerable sets is a denumerable set.

(ii) The set of all rational numbers is denumerable. 4+4

(b) If f is a similarity mapping of the well-ordered set X onto the subset $Y \subseteq X$, then show that $x \leq f_{(x)}$ for all $x \in X$. 8

2. (c) Prove the following :

(i) $\aleph_0 \aleph_0 = \aleph_0$

(ii) $\aleph_0 C = C$

When $\aleph_0$ is the cardinality of a denumerable set and C is the cardinality of the set of all real numbers. 8

(d) Show that multiplication of order types is not commutative. 8

**UNIT—II**

3. (a) Define :

(i) Topological Space (ii) Limit point

(iii) Interior of a set (iv) Neighborhood of a point. 8

(b) Define relative topology. Prove that $\tau^* = \{G \cap X^* : G \in J\}$ is a topology for $X^*$, where $X^* \subseteq X$ and $(X,\tau)$ is a topological space. 2+6

4. (c) Show that the union of two topologies for a set need not be a topology for the set, but the intersection of any family of topologies for a set will be a topology for that set. 2+6

(d) Define closed set.

If $x \notin F$, Where F is a closed subset of a topological space $(X,\tau)$, then show that there exists an open set G such that $x \in G \subseteq CF$. 8

**UNIT—III**

5. (a) If a connected set C has a nonempty intersection with both E and the complement of E in a topological space $(X,\tau)$, then prove that C has a nonempty intersection with the boundary of E. 8

(b) Define open mapping.

Show that a mapping f of X into X* is open iff $f(i(E)) \subseteq i^*(f(E))$ for every $E \subseteq X$. 8

6. (c) Prove that a topological space $(x,\tau)$ is compact iff any family of closed sets having the finite intersection property has a nonempty intersection. 10

(d) If f is a continuous mapping of $(X,\tau)$ into $(X^*, \tau^*)$ then show that f maps every compact subset of X onto a compact subset of X*. 6

## UNIT—IV

7. (a) Show that in a Hausdorff space, a convergent sequence has a unique limit. 6

(b) Prove that if f is a mapping of the first axiom space X into the topological space X*, then f is continuous at $x \in X$ iff for every sequence $<x_n>$ of points in X converging to x we have the sequence $<f(x_n)>$ converges to the point $f(x) \in X^*$. 10

8. (c) Show that a topological space X is a To-space iff the closures of distinct points are distinct. 8

(d) Prove that in a second axiom space, every open covering of a subset is reducible to a countable subcovering. 8

## UNIT—V

9. (a) Define :

(i) Regular Space

(ii) Normal Space

(iii) Completely Normal

(iv) Completely Regular. 8

(b) Show that a topological space X is normal iff for any closed set F and open set G containing F, there exists an open set G* such that $F \subseteq G^*$ and $(G^*) \subseteq G$. 8

10. (c) Show that a normal space is completely regular iff it is regular. 8

(d) Prove that regularity is a topological property. 8