First Semester M. Sc. (Part – I) C.B.C.S. Scheme Examination

PHYSICS

Mathematical Physics

Paper – 1 PHY 1

P. Pages : 5

Time : Three Hours] [Max. Marks : 80
[Credit : 04

Note : All questions are compulsory and carry equal marks.

1. (A) Show that : $(AB)^{-1} = B^{-1} A^{-1}$. 5

(B) Define orthogonal matrix. Show that the matrix $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is an orthogonal. 7

(C) Find the rank of the matrix $A = \begin{pmatrix} 3 & 4 & 3 \\ 9 & 12 & 3 \\ 3 & 4 & 1 \end{pmatrix}$ 4

OR

(p) Show that eigen values are invariant under similarity transformation. 5

(q) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ 7

(r) If A is an orthogonal matrix then show that $|A| = \pm 1$. 4

2. (a) Expand the function $\log(1+z)$ about $z=0$ in Taylor's series. 4

(b) If $u(x, y) = x^2 - y^2$ is the real part of an analytic function $f(z) = u + iv$, find v. 4

(c) State and prove Cauchy's integral formula. 8

OR

(p) Expand the function $f(z) = \dfrac{1}{z(z-1)}$ about $z=0$ and $z=1$ in terms of Laplace's series. 4

(q) Find the residue of $f(z) = \dfrac{ze^z}{(z-a)^3}$. 4

(r) State and prove Cauchy's residue theorem. 8

3. (a) Show that the orthogonality condition for Legendre polynomial

$$\int_{-1}^{1} P_m(n) \cdot P_n(n) = 0 \text{ if } m \neq n.$$ 8

(b) Prove the following recurrence relations

(i) $(2n+1)\,x\,P_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$

(ii) $nP_n(x) = xP'_n(x) - P'_{n-1}(x)$ 8

OR

(p) State and prove the Rodrigue's formula for Legendre's polynomial. 8

(q) Show that $(1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)\,t^n$ is the generating function of Legendre polynomial. 8

4. (a) Prove the following recurrence relations for Hermite polynomials :

$$2x\,H_n(x) = 2n\,H_{n-1}(x) + H_{n+1}(x)$$ 4

(b) Show that $e^{2xt-t} = \sum_{n=0}^{\infty} \dfrac{H_n(x)}{n!} t^n$. 8

(c) Prove that $x\,J'_n(x) = x\,J_{n-1}(x) - nJ_n(x)$. 4

OR

(p) Show that

$$H_n(x) = (-1)^n \exp(x^2) \cdot \frac{d^n}{dx^n} [\exp(-x)^2].$$ 4

(q) Show that $\exp\left[\frac{1}{2} x \left(1 - \frac{1}{t}\right)\right] = \sum_{n=-\infty}^{\infty} J_n(x) t^n.$ 8

(r) Prove that $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x).$ 4

5. (a) Find Fourier transform of $f(x) = 1/x$. 4

(b) Define Laplace transform for the function F(t). Find Laplace transform for the function e^{at} if $s > a$. 4

(c) Define Fourier series and obtain a_0, a_n and b_n coefficients. 8

OR

(p) Find inverse Laplace transform for the function $f(s) = \left[\frac{1}{s-2} + \frac{2}{s+5} + \frac{6}{s^4}\right]$. 4

(q) Obtain Fourier transform of

$$f(x) = \begin{cases} 1; & |x| < a \\ 0; & |x| > a \end{cases}$$ 4

(r) If f(s) is the Fourier transform of F(x), then show that $e^{-ias} \cdot f(s)$ is the Fourier transform of $F(x-a)$. 4

(s) Define Fourier transform for the function F(x) in the interval $-\infty < x < \infty$. If f(s) is the Fourier transform of f(x) then show that the Fourier transform of $F'(x)$ is $isf(s)$ if $F(x) \rightarrow 0$ as $x \rightarrow \pm\infty$. 4