AU–222

M.Sc. Semester—I (C.B.C.S. Scheme) Examination

PHYSICS

1-Phy-1 : Mathematical Physics

Time : Three Hours] [Maximum Marks : 80

Note :—All questions are compulsory and carry equal marks.

EITHER

1. (A) Find the rank of a matrix :

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}.$$ 4

(B) Determine whether the 4-vectors (1, 2, 3), (2, 0, –1), (1, –1, 1) and (2, 1, 0) are linearly dependant or independant. 4

(C) Find the characteristic equation of matrix :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

and verify that it is satisfied by A, hence find the inverse of A. 8

OR

(P) If A is hermitian matrix then show that B^*AB is hermitian for every matrix B. 4

(Q) Show that every square matrix can be uniquely expressed as the sum of a symmetric and skew symmetric matrix. 4

(R) Find the eigen values and eigen vectors of matrix :

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$ 8

EITHER

2. (A) Test the function $w = e^z$ for its analyticity in fourth quadrant. 4

(B) Expand $f(z) = \dfrac{1}{(z+1)(z+3)}$ in Laurent series, which is valid for $1 < |z| < 3$. 4

(C) If $f(z)$ is analytic inside and on a simple close curve C and if z_0 is any point inside C then show that

$$f(z_0) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z - z_0)} dz. \qquad 8$$

OR

(P) Show that

$$\int_0^{2\pi} \frac{d\theta}{\frac{5}{4} + \sin\theta} = \frac{8\pi}{3}. \qquad 4$$

(Q) Explain function $\log(1+z)$ about $z = 0$ in Taylor's series. 4

(R) Show that $u = y^3 - 3x^2y$ is a harmonic. Find its harmonic conjugate and corresponding analytic function. 8

EITHER

3. (A) State the Legendre's equation. Derive Legendre's polynomial of second kind. 10

(B) Show that $P_n(-x) = (-1)^n P_n(x)$. 6

OR

(P) State and Prove Rodrique's formula for Legendre's polynomial. 8

(Q) Prove the following recurrence relations :

(i) $nP_n = x P'_n - P'_{n-1}$ 4

(ii) $P'_{n+1} - P'_{n-1} = (2n+1) P_n$. 4

EITHER

4. (A) Show that :

$$e^{2xt - t^2} = \sum_{n=0}^{\infty} \frac{H_n(x)}{n!} t^n. \qquad 8$$

(B) Establish the relation :

$$J_n(x)\cdot J'_{-n}(x) - J'_n(x)\cdot J_{-n}(x) = -\frac{2\sin n\pi}{\pi x}.$$ 8

OR

(P) Show that

$$\exp\left[\frac{1}{2}x\left(1-\frac{1}{t}\right)\right] = \sum_{n=-\infty}^{\infty} J_n(x)\, t^n.$$ 8

(Q) Prove that

(i) $2n\, H_{n-1}(x) = H'_n(x)$ 4

(ii) $2x\, H_n(x) = 2n\, H_{n-1}(x) + H_{n+1}(x)$. 4

EITHER

5. (A) Find the Fourier transform of

$$F(t) = e^{-t^2/2}.$$ 6

(B) Using convolution theorem, show that :

$$L^{-1}\left[\frac{1}{s(s^2+a^2)}\right] = \frac{1}{a^2}(1-\cos at).$$ 4

(C) Obtain the Fourier series of the function

$$F(x) = x \sin x \quad ; -\pi < x < \pi.$$ 6

OR

(P) State and prove initial value theorem. 6

(Q) Find inverse Laplace transform of

$$\frac{s+2}{s^2-4s+13}.$$ 4

(R) Find the Laplace transform of

$$F(t) = \begin{cases} \sin t & \text{for } 0 < t < \pi \\ 0 & \text{for } t > \pi \end{cases}$$ 6