M.Sc. (Part–I) Semester–I (CBCS Scheme) Examination

PHYSICS

Paper : 1-PHY-1

(Mathematical Physics)

Time : Three Hours] [Maximum Marks : 80

Note :— ALL questions are compulsory and carry equal marks.

EITHER

1. (A) Show that $\alpha_x\beta + \beta\alpha_x = 0$,

where $\alpha_x = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$. 4

(B) Show that the matrix $A = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{2} \\ 1/\sqrt{3} & -\sqrt{2}/3 & 0 \\ 1/\sqrt{3} & 1/\sqrt{6} & -1/\sqrt{2} \end{bmatrix}$ is an orthogonal and obtain its inverse. 5

(C) Find the inverse of linear transformation

$u = -2x - 2y + 7z$

$v = 4x + 3y - 12z$

$w = -x + 2z$

by obtaining the inverse of matrix of transformation. 7

OR

(P) Show that the matrix $A = \begin{bmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ -i/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$ is unitary. 3

(Q) Show that two eigen vectors of Hermitian matrix having different eigen values are orthogonal. 4

(R) Reduce the matrix $A = \begin{pmatrix} 5 & 7 & -5 \\ 0 & 4 & -1 \\ 2 & 8 & -3 \end{pmatrix}$ to a diagonal form. 9

EITHER

2. (A) Evaluate $\oint_c \frac{3z^2+z}{z^2-1}\,dz$

where c is circle $|z| = 2$. 4

(B) Expand $f(x) = \frac{1}{(z-1)(z-2)}$, which is valid for $1 < |z| < 2$. 4

(C) Define isolated singular point.

Show that $\oint_c f(z)\,dz = 2\pi i \sum_{k=1}^{n} [\text{Res}\, f(z)]_{z=z_k}$ 8

OR

(P) Evaluate $\oint_c \frac{dz}{z^3(z-4)}$ taken counter clockwise around the circle $|z+2| = 3$. 4

(Q) Test the function $w = \log z$ for its analyticity. 4

(R) Define singularity. Prove the Cauchy's integral theorem. 8

EITHER

3. (A) State the Legendre's equation and derive Legendre's polynomial of first kind. 8

(B) Prove that :

(i) $\int_{-1}^{+1} P_0(x)\,dx = 2$ 2

(ii) $\int_{-1}^{+1} P_n(x)\,dx = 0\ (n \neq 0)$ 6

by using Rodrique formula.

OR

(P) Show that :

$$\int_{-1}^{+1} P_m(x)\,P_n(x)\,dx = 0 \text{ if } m \neq n$$

$$= \frac{2}{2n+1} \text{ if } m = n$$ 10

(Q) Prove the recurrence relation

$nP_n = (2n - 1)\ xP_{n-1} - (n - 1)\ P_{n-2}$ 6

EITHER

4. (A) Prove that the generating function of Hermite polynomials is

$$\exp\{x^2 - (\rho - x)^2\} = \sum_{n=0}^{\infty} \frac{H_n(x)}{n!} \rho^n.$$ 8

(B) Find the values of :

(i) $J_{1/2}(x)$

(ii) $J_{3/2}(x)$ 4+4

OR

(P) Using the generating function of Hermite polynomial $H_n(x)$. Prove that

$$\frac{1}{e}\cosh 2x = \sum_{n=0}^{\infty}\left(\frac{1}{(2n)!}\right)H_{2n}(x).$$ 8

(Q) Show that the Bessel's function $J_n(x)$ of the first kind of order 'n' is the coefficient of t^n in the expansion of the function $\exp\left[\frac{x}{2}\left(t-\frac{1}{t}\right)\right]$ i/e $\exp\left[\frac{x}{2}\left(t-\frac{1}{t}\right)\right] = \sum_{n=-\infty}^{+\infty} J_n(x)\, t^n$. 8

EITHER

5. (A) Find Laplace transform of $e^{-st} . \sin 3t$. 6

(B) State and prove convolution theorem of Fourier transform. 6

(C) Find inverse Laplace transform of :

$$f(s) = \left[\frac{1}{s-2} + \frac{2}{s+5} + \frac{6}{s^4}\right].$$ 4

OR

(P) State and prove final value theorem. 6

(Q) Find Fourier series of function $f(x) = x^2$ for $0 < x < \pi/2$. 4

(R) Find a cosine series for $f(x) = \begin{cases} x & , \ 0 < x < \pi/2 \\ \pi - x & , \ \pi/2 < x < \pi \end{cases}$. 6