(b) Define completeness and bounded completeness. Show that sample mean is a complete Statistic for μ for normal population with known mean. (8+8)

OR

(B) (i) State and prove Lehman–Sheffe theorem.

(ii) Explain the term 'Blackwelization'. Discuss its use in inference. (8+8)

5. (A) (a) Elaborate the following terms with example :—

(i) Confidence coefficient.

(ii) Degree of confidence.

(iii) Length of confidence interval.

(b) Construct $(1-\alpha)$ 100% confidence interval for μ on the basis a random sample from N (μ, σ^2) where σ^2 is known. (8+8)

OR

(B) (i) Define :—

(a) Pivot.

AQ – 832

First Semester M. A. / M. Sc. (Part–I) Examination

STATISTICS

1 SCA – 2

(Estimation Theory)

P. Pages : 5

Time : Three Hours] [Max. Marks : 80

Note : Solve either (A) or (B) from each question.

1. (A) (a) Define unbiased estimator. Show that sometimes they are absurd. Also show that they need not be unique.

(b) Explain the concept of likelihood function. Obtain likelihood function for the following distributions.

(i) Normal distribution

(ii) Poisson distribution. (8+8)

OR

(B) (i) Explain the concept of consistency in estimation theory. Also establish its invariance property.

(ii) Define minimum variance unbiased estimator (MVUE). If T_1 is MVUE of parameter θ and T_2 is any other unbiased estimator of θ then show that no linear combination of T_1 and T_2 can be MVUE of θ. (6+10)

2. (A) (a) State only Cramer – Huzurbazar theorem along with the regularity conditions.

(b) Explain with example method of moments for estimating the parameter.

(c) Show that MLE need not be unique. (4+6+6)

OR

(B) (i) Define Cramer family. Does family of Cauchy distribution belong to this family.

(ii) Explain the procedure of obtaining MLE by the method of scoring.

(iii) Obtain MLE of σ^2 on the basis of a r. s. taken from normal distribution with unknown mean μ and variance σ^2. (4+6+6)

3. (A) (a) Explain :—

(i) Fisher Information.

(ii) Fisher Information matrix.

Discuss its role in statistical inference.

(b) Under certain regularity conditions (to be stated) establish the CRLB for the variance of an unbiased estimator of some parametric function. (6+10)

OR

(B) (i) Explain with example :—

(a) Exponential family of density.

(b) Pitmen family of density.

(ii) Define :—

(a) Sufficient statistics.

(b) Minimal sfficient statistic.

Show that number of successes in n independent Bernoulli trials is sufficient for θ, where θ is probability of success in a single trial. (6+10)

4. (A) (a) State and prove Rao–Blackwell theorem. Discuss its applications.

(b) Shortest length confidence interval.

(c) Shortest expected length confidence Interval.

(ii) Construct $(1 - \alpha)$ 100% shortest length confidence interval for μ when σ^2 is known. (6+10)