AU–247

M.A./M.Sc. (Part–I) Semester–I (C.B.C.S. Scheme) Examination

STATISTICS

(Estimation Theory)

Paper—II

Time : Three Hours] [Maximum Marks : 80

N.B. :— Solve either A **or** B part from each question.

1. (A) (a) Explain :

(i) Consistency (ii) Efficiency

(iii) MVUE (iv) Likelihood function 8

(b) If T_1 is MVUE of θ and T_2 is any other unbiased estimator of θ with efficiency less than 1, then no unbiased linear combination of T_1 and T_2 can be MVUE of θ. 8

OR

(B) (i) Explain unbiased estimator and show that sometimes it is absurd. 6

(ii) Prove that correlation coefficient between MVUE and another unbiased estimator is $\rho = \sqrt{E}$ where E is efficiency of unbiased estimator. 10

2. (A) (a) Describe the method of maximum likelihood. Show that MLE need not be unbiased. 8

(b) Suppose $x_1, x_2, \ldots, x_n$ be a r.s. from Cauchy population with pdf $\frac{1}{\pi[1+(x-\theta)^2]}$.

Estimate the parameter θ by method of scoring. 8

OR

(B) (i) Describe method of moment to estimate the parameter θ. Estimate the parameter μ and σ of Normal distribution by this method. 10

(ii) State only Cramer Huzurbazar theorem along with all the regularity conditions. 6

3. (A) (a) Let $x_1, x_2, \ldots, x_n$ be a r.s. of size n from normal population with mean μ and variance θ, where θ is unknown. Find CRLB for θ. 8

(b) State factorization theorem. Let $x_1, x_2, \ldots, x_n$ be a r.s. from normal population with mean θ_1 and variance θ_2, then obtain sufficient statistic for θ_1 and θ_2. 8

OR

(B) (i) State and prove Cramer Rao inequality. 10

(ii) Explain single and multi parameter exponential family. 6

4. (A) (a) State and prove Rao Blackwell theorem. 10

(b) Define the term :

(i) Completeness

(ii) Bounded completeness. 6

OR

(B) (i) Let X be a r.v. with

$$P_{\theta}(x) = \theta^2(1-\theta)^x \; ; x = 0, 1, 2, \ldots$$
$$= 1-\theta \qquad ; x = -1, 0 < \theta < 1$$

Show that this family of distribution is not complete. 8

(ii) State and prove Lehman Scheff theorem. 8

5. (A) (a) Define the following term :

(i) Shortest length C.I.

(ii) Shortest expected length C.I. 6

(b) Let $x_1, x_2, \ldots, x_n$ be a r.s. of size n from $N(\mu, \sigma^2)$. Find shortest length confidence interval for μ when :

(i) σ^2 is known

(ii) σ^2 is unknown 10

OR

(B) (i) Define :

(a) Pivot

(b) Confidence coefficient

(c) Degree of confidence interval. 6

(ii) Construct $(1-\alpha)$ 100% confidence interval for σ^2 on the basis of r.s. from $N(\mu, \sigma^2)$ when μ is known. 10